The Africa Regional Congress of ICMI on Mathematical Education (AFRICME 5)

Quality Mathematics Education for All

August 29-31, 2018

Foreword

The Aga Khan University, Institute for Educational Development, East Africa in collaboration with the International Commission on Mathematical Instruction (ICMI) has the pleasure of sponsoring, *The Fifth Africa Regional Congress of ICMI on Mathematical Education (AFRICME 5)*. The event takes place at the Aga Khan University campus in Dar es Salaam,Tanzania from Aug 29- 31, 2018.

The AFRICME 5 programme offers an exciting combination of keynote lectures, panel and round table discussion, paper presentations, workshops, regional presentations, displays of local teaching learning materials and school visits. Focus in all these will be on key developments and research in Mathematics Education particularly in Sub-Saharan Africa. The theme and sub-themes are as follows:

Theme

Quality Mathematics Education for All

Sub-themes

- Effective initial and continuing Mathematics Teacher Education
- Inclusion and equity in Mathematics Education (gender, multilingualism, special needs)
- Mathematics knowledge in and for teaching
- Integrating ICT in Mathematical Education
- Mathematical thinking for nurturing quality education
- Assessment and evaluation issues in Mathematics Education
- The role of contextually relevant research in quality Mathematics Education

The present volume is a compilation of the research papers submitted on a recently completed or an ongoing study. While the abstracts are carefully reviewed to ensure quality and relevance of the presentation, the papers are not peer-reviewed. The papers presented do not represent the views of the sponsors and their partners or the Conference Organizing Committees.

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Chair AFRICME 5

Dr. Fredrick Mtenzi
Co-chair AFRICME 5

Aga Khan University,Institute for Educational Development,East Africa
August 2018

Acknowledgement

Conferences and events such as the AFRICME 5 are first a team effort than anything else! We thank all those who have made possible this conference and the compilation of this current volume of papers.

The sponsors, Aga Khan University Institute for Educational Development, East Africa and the International Commission on Mathematical Instruction generously supported the AFRICME 5 in a number of ways including funds. Without their generosity it would not have been possible to organize the AFRICME 5 in Tanzania.

The programme of AFRICME 5 was strengthened by the contributions of the keynote speaker Professor Barbara Jaworski, the workshop leaders and the panellists. We are grateful to them for their time and scholarly contributions.

The administrative team and the office of public affairs at the Aga Khan University worked tirelessly and diligently to ensure that the conference was efficiently organized. Their support in the background made the academic deliberations possible. Thank you !

Members of the Local Organizing Committee and the International Advisory Committee, listed below gave generously of their time to ensure the quality and relevance of the AFRICME5 programme. We are deeply indebted to them for their generosity.

International Advisory Committee

Local Organizing Committee

Professor Anjum Halai
Chair AFRICME 5

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Keynote address

Teaching mathematics with sensitivity and challenge

Barbara Jaworski
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This paper addresses the achievement of an inquiry-based mathematical learning environment that provides for each individual student within a 'pedagogy for all' and works overtly to accommodate difference. Three narratives from research set the scene for the kind of activity that is envisaged, followed by discussion of theoretical perspectives that underpin such activity. Research is located within a sociocultural perspective in which 'communities of inquiry' are a central feature. A tool, the 'teaching triad', is proposed both to analyse teaching and support its development. Finally, the paper offers examples from a research project, based on the teaching triad, which aimed to develop inquiry-based activity in classroom settings in collaboration between teachers and educators. Issues arising provided important learning experiences for teachers and educators.

Keywords: Mathematics learning and teaching; community of inquiry; inquiry-based practice; developmental research and practice; the Teaching Triad.

INTRODUCTION

This paper focuses on mathematics teaching practices with a special consideration for diversity and difference in the classroom. Sometimes, the issues of diversity and difference are addressed within an umbrella heading of "special needs" and how we cater for special needs in the classroom. Since teaching has the specific purpose of creating learning (Pring 2004), I want to make the point that learning can only take place if a teacher pays sensitive attention to the needs of students from affective, cognitive, social and cultural perspectives.

Special for all, special for one – a pedagogy for all

Using the terminology of special needs, I would like to suggest a focus of "Special for all, special for one", with the *all* coming before the *one*. I contrast this with an alternative, "special for one, special for all", which might be seen as a more common phrase. Putting the *all* first is extremely important in designing mathematics teaching which creates opportunity for all students at any level, whatever their needs, to engage successfully with mathematics. Creating such opportunity in classrooms is a challenge for a mathematics teacher and for the development of teaching that seeks such provision[1].

[1] I draw partially on a presentation I gave at the NORSMA 5 conference in Reykjavik in 2009 -- the Nordic Research network on Special Needs Education in Mathematics http://stofnanir.hi.is/norsma/

In the matter of *achievement for the one* within *provision for the all* the idea of difference is important, for example the differences that we see in individual learners: differences in gender, race, social class, neurodiversity, physical diversity, mathematical anxiety. Such differences contribute to a diverse mathematics classroom setting in which it is important to include the individual within the all. Two ways of seeing this relationship are:

- Special for each one: we attend to the special needs of each student, and therefore make the situation special for all.
- Special for all: we attend to the special needs of the whole group and hence make the situation special – provide opportunity – for each one in the group.

The second of these suggests a pedagogy for all, and inclusion of all, that deals with difference and diversity. Research into neurodiversity in education – including differences like dyslexia, dyspraxia, dyscalculia, autism and ADHD– raises issues about how education is made inclusive for all of these 'special' needs. Pollack (2009, p. 7) writes " are we not all neurodiverse?" He writes further

> If the goal of inclusion is to be attained, it will only be [attained] by considering the specifics of need as well as a pedagogy for all. (Pollak, p.7, citing Powell 2003, p. 6)

A key question here concerns how we make the distinction; *how* we provide for difference within a pedagogy for all.

For example, differences in students' cognitive style may be seen in Chinn & Ashcroft's inchworms and grasshoppers: *inchworms* focus on parts and details, they separate ideas; *grasshoppers* tend to overview, to the holistic, putting ideas together. They write also:

> You can usually go a long way to finding out how a child solves a problem by asking the simple question 'How did you do that?' This interest, based on awareness rather than a judgment, will be a major source of help for many students, especially when combined with an awareness of what the child brings to the question (Chinn & Ashcroft,1998, p. 23-4)

It is well known that many students at all levels experience difficulties with mathematics (Cockcroft, 1982) and that these difficulties create anxiety for the student (Tobias, 1993), perhaps from seeing the difficulties as their own fault, due to their own problems with the subject, problems not experienced by other students and not recognised by their teacher. The following quotation comes from a student, Jane, recorded by my colleague Clare Trott[2] (with Jane's permission) as part of her research on special needs experienced by students who came to her for help.

> The feeling that I had when I was learning GCSE maths was very difficult. I couldn't understand the concept of the numbers between nought and one; I couldn't understand the fractions. It may seem very simple to people who are looking at this now, but for me it was

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[2] Clare Trott directs the *Eureka Centre for Mathematical Confidence* at Loughborough University in which she works with additional needs students including those diagnosed as dyslexic or dyscalculic (e.g., Trott, 2008).

> very difficult, and it caused me a lot of anxiety. I was trying to learn the maths and the anxiety got involved … it hampered my learning. I had to control the anxiety as well as learn the maths. … I felt a lot of frustration. It's unfair that I can't understand these basic maths, I should be able to, but I just couldn't do it.

How can a teacher at any level provide the kind of environment and opportunities that enable someone like Jane to learn mathematics, and how might this be done as part of a pedagogy for all?

Within the mathematics education literature there is wide consideration of issues of gender, race and social class as they pertain to mathematics teaching and learning. Girls are seen often to have different approaches to learning than boys, and teachers have been seen to treat girls and boys differently in the classroom (e.g. Burton et al. 19xx). Social class has been seen to affect considerably the ways in which students respond to standard assessment tasks relating to the mathematics curriculum (e.g. Cooper and Dunne, 200x). Racial or ethnic groups have been seen to bring differing ways of doing and seeing mathematics (e.g., , and multilingualism raises issues for language and symbolisation in mathematics (e.g., .

As soon as we start to enumerate the many sorts of difference and the issues they raise potentially for teachers and students in the classroom, the challenges for the teacher can seem enormous. What can something called "a pedagogy for all" look like, which strives to include all students in relation to their needs? Well, I am not going to present a magic formula or blueprint, but I do want to suggest ways of approaching these challenges.

SOME GUIDANCE FROM THE LITERATURE

Starting from a Vygotskian position that all learning is social and that individuals internalize from their engagement in sociohistorically rooted settings, the difficulties that learners experience in mathematics can be seen to relate to ways in which society, schooling, and mathematics have taken on their current characteristics (Daniels, 2001; Vygotsky, 1978). For example, in my own country, England, schools organize students into ranked sets based on their mathematical achievement (Boaler and Wiliam, 2001); it is common for people to acknowledge that mathematics was their worst subject at school and that they found it boring and intractable (Cockcroft, 1982). Setting achieves a broad separation of students according to their ability to work within the system and, as Boaler and Wiliam show, results in alienation from mathematics for many. The quotation from Jane above shows just one example. Nardi and Steward (2003) report from typical English classrooms that classroom mathematics is T.I.R.E.D; their study shows that students who are 'quietly' disaffected exhibit characteristics of *t*edium, *i*solation, *r*ote learning, perceptions such as *e*litism and *d*epersonalization from mathematics. Thus, the systems of setting and approaches to teaching mathematics that are common in English classrooms are ignoring diverse needs and alienating a wide range of students.

The literature suggests that inclusive approaches to learning and teaching mathematics need to respect and celebrate diversity, recognize and value difference, maintain awareness of anxiety, and challenge students appropriately (e.g., Chin & Ashcroft, 1998; Tobias, 1993). Ollerton & Watson (2001) draw attention to the 1999 National Curriculum for England in Mathematics which sets out "three principles for the development of an inclusive curriculum: setting suitable challenges, responding to diverse needs and overcoming potential barriers to learning and assessment" (p. 3). Ollerton and Watson go on to say, "Given that a mathematics qualification is an important passport to higher education and further social and economic opportunity, it is especially important that mathematics teachers do not limit the possibilities for their students."

Skovsmose and Säljö (2008), writing from Scandinavian experiences, refer to "an exercise paradigm" as dominant in the culture of mathematics classrooms widely and limiting opportunity (p. 40). They write:

> This [the *exercise paradigm*] implies that the activities engaged in the classroom to a large extent involve struggling with pre-formulated exercises that get their meaning through what the teacher has just lectured about. An exercise traditionally has one, and only one, correct answer, and finding this answer will steer the whole cycle of classroom activities and the obligations of the partners involved … (p. 40).

In contrast to the exercise paradigm they propose that a focus on mathematical inquiry open up possibilities:

> The ambition of promoting mathematical inquiry can be seen as a general expression of the idea that there are many educational possibilities to be explored beyond the exercise paradigm (p. 40).

For educators, teachers and school organizers, therefore, it seems important to address what it means *to include, to recognize and value, to respect and celebrate, to challenge all students at appropriate levels and to approach mathematics through inquiry*. How is a teacher in any setting at any level to achieve such provision? The next section includes three examples, three narratives drawn from studies of classroom data in which I have been involved, that illuminate these questions and suggest a basis for a pedagogy for all that celebrates diversity.

EXAMPLES OF ADDRESSING DIVERSITY IN MATHEMATICS CLASSROOMS

Turning "I can't" into "I can and I did"

This narrative comes from a project conducted jointly between the Open University and the Mathematical Association in the UK in the early 1980s: *Working Mathematically with Low Attainers*. Mathematics educators from the Open University worked with teachers in a number of schools to focus on ways of teaching low attaining students to enable mathematical achievement. A published videotape (Open University, 1985) resulting from the project had the title "Turning I can't into I can and I did".

In this video compilation we see a teacher, working with a class of students on the problem "If a number of circles intersect in a plane, how many regions can be created?". The teacher had taken into the classroom a set of "hoola hoops" (large, plastic, brightly coloured hoops, which she and the students used to represent circles in real space. Some students used the hoops, others drew circles on the board or in their books, arranging their circles to try to find the maximum number of regions for a given number of circles. In each case they counted regions and noted down their results: one circle, one region; two circles, three regions, three circles, seven regions,

Table 1: Numbers of hoops and regions

Hoops	Max. Regions
1	1
2	3
3	7
4	13
5	?
…	…

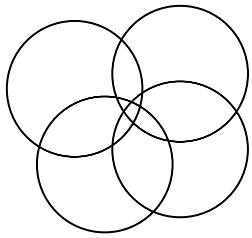

Figure 1 – Maximum number of regions with 4 circles

The case of 4 hoops is shown. Regions inside the hoops may be counted to reveal 13. An important part of the mathematics here is to justify that this is the maximum number of regions for 4 circles and to relate this to the number of circles more generally. Most students had addressed such questions and come up with convincing explanations, and the teacher had encouraged them to express their findings algebraically. Then a student, Mary, approached, and showed the teacher the work in her book. She had drawn the following diagram (Figure 2)

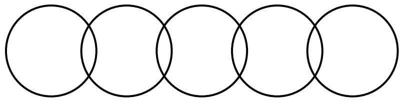

Figure 2: Mary's arrangement of circles

And her table was as follows:

Table 2: Mary's results

Hoops	Max. Regions
1	1
2	3
3	5
4	7
5	9
…	…

The teacher asked Mary to explain what she had done and what she had found, and seemed satisfied with Mary's response. Then finally the teacher said:

"You're doing something different from everyone else, Mary. Don't worry, that's fine. Just ignore what everyone else is doing".

She asked Mary to write an algebraic formulation of her own case.

All students here had the opportunity to decide for themselves how to tackle the problem, with the teacher encouraging and challenging them to seek mathematical generalization.

This situation *for all* allowed Mary to do things differently. The teacher had to balance a wish to respect and value Mary's work with a wish for Mary to see a broader generality in the situation. The investigative situation allowed for such differences of approach and direction, but also raised questions about overall achievement in terms of the required curriculum and desired mathematical achievement. The teachers had to address these issues in deciding to respond in this way to Mary.

Investigating mathematics teaching – a lesson on fractions

The second narrative comes from my own research (Jaworski, 1994, p. 92). A teacher Clare, was working with her whole class on *fractions*. She pointed to ½ (written on the board) and asked one girl, Katy, "What is one divided by 2?". Katy said "two". Clare asked Katy to work out on her calculator, "one divided by 2". Katy said "Nought point five". Clare asked, "Surprised?" "If you have one thing shared between two people, how much does each get?" Katy looked blank.

Clare then said to the whole class: "Can we have a 'hands-down-think' (students were used to being asked to *think* without waving their hand for attention). I did ½. I want you to think what you might do next." Then she went to talk with Katy.

Later in the lesson she was working with the whole class on 1/7 – dividing 1 by 7 to get a decimal representation – some students were finding this problematic. To the others in the class she said "Anyone who's ahead of this, try to think how to explain repetition in 1/7" [1/7 = 0.142857142857142…].

The class was a mixed ability class in Year 8 (ages 12-13). Katy was one of a number of students who were struggling with the work on fractions, although there were other students in the class who could tackle more challenging questions. We see here a teacher differentiating according to the needs of different groups of students. The emphasis on *thinking* was typical of her situation; *all* students were required to *think* (before putting up their hands to give an answer). Those who understood the more basic questions were given more challenging problems to think about; *explaining* repetition in 1/7 is seriously challenging for students of this age. Students had to take some responsibility for which group to be in, whether to work further on the basic ideas or to tackle the more difficult problem. Thus the teacher encouraged students to think not only about their mathematics but about their degree of understanding. They learned to make choices within what was offered in the classroom. It was a challenge for the teacher to maintain levels of achievement commensurate with students' abilities and needs and in many cases this raised issues for teaching.

What shape is it?

This narrative comes again from my own research (Jaworski, 1988, p. 287). Look at the drawing in Figure 3. What is it? What shape is it?

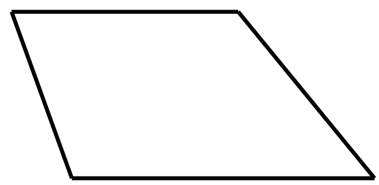

Figure 3: The teacher's original drawing

A Year 8 class had been asked by their teacher to name the shape, which he had drawn on the board. Someone said that it was a trapezium. Some students agreed with this, but not all. The teacher said, 'If you think it's not a trapezium then what is it?' Michael said, tentatively, 'It's a square …'. There were murmurings, giggles, 'a square'?! … But Michael went on '… sort of flat.'

The teacher looked puzzled, as if he could not see a square either. He invited Michael to come out to the board and explain his square. Michael did. He indicated that you had to be looking down on the square – as if it were on your book, only tilted. He moved his hand to illustrate. 'Oh' said the teacher. 'Oh, I think I see what you mean … does anyone else see what he means?' There were more murmurings, puzzled looks, tentative nods.

Then the teacher drew onto the original shape as in Figure 4.

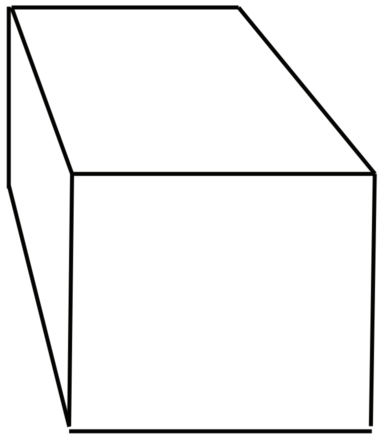

Figure 4: The teacher's modified drawing

Oooooh yes (!) said the students and there were nods around the class.

Here the teacher had to be prepared to suspend his initial plan for the lesson to accommodate Michael's special viewpoint. The result showed an enhanced vision for the whole class, special encouragement for Michael, and a contribution to an ethos of listening to and respect for others. We can see this as a moment of 'contingency'(Rowland et al, 200x) in which the teacher departs from his planning for the lesson to encourage a new idea and use it to effect for the whole class.

Tasks for all

Each of these teachers had put care and thought into their design of tasks for students. In each case, the nature of the task and the way the teacher worked with the students allowed everyone to make a start, diverse directions and ways of thinking, fluidity and flexibility in activity and serious mathematical thinking and outcome. The teachers' actions encouraged all students to participate, supported individuals who do things differently, provided extra support where it was needed and challenged all students mathematically at appropriate levels. We saw in each case a certain degree of *contingency* (Rowland, Huckstep & Thwaites, 2005), in which the teachers needed to respond and make decisions in the moment as to how to act. My observations in each classroom over a period of time suggested that these were not just serendipitous moments but were a result of careful ethos building over considerable time. Environments in which such contingencies arise do not happen by chance or overnight; they need to be worked at overtly and nurtured by the teacher. Teachers also need visions of mathematics as an open and flexible subject in which all students can participate and in which challenges can be offered to deal with widespread needs. An inclusive classroom environment can enable student choice and responsibility, allow for the teacher to recognize and respond to needs as they arise and to work with them over time.

Demands on a teacher

The narratives highlight the complex demands on a teacher of creation of an environment that is inclusive and respects diversity. This complexity includes:

- *a teacher's own knowledge, confidence and love of mathematics*
- *a teacher's design of tasks that encourage participation, connection and understanding in mathematics*
- *a teacher's use of resources in ways that support learning*
- *a teacher's knowledge of students and their particular needs*

Teachers have to be knowledgeable and experienced in mathematics, having a vision of where what they teach is going: for example, teaching pattern spotting with algebra in mind; teaching fractions with rational numbers in mind; teaching 2-dimensional geometry with three dimensions in mind. This requires them to act in didactic mode – that is in a mode of converting their own mathematical understanding into tasks for students in which students can have opportunity to reach mathematical understanding. In addition, teachers have to have a vision of classroom interaction in mathematics which allows difference and diversity to flourish. This requires pedagogic understanding and a knowledge and vision of strategies that can engage students and encourage participation in and understanding of mathematics.

Thus, what is needed is a bringing together of the mathematics, the didactics and the pedagogy in a way that respects and celebrates diversity, recognizes and values difference, and includes everyone. How do we go about achieving these very serious demands? How does this knowledge and these qualities develop? Is this demanding too much of teachers? In the next section, I will introduce the idea of *inquiry* as a central concept for classroom didactics and pedagogy and a theoretical tool, the teaching triad, for analyzing and developing teaching.

AN INQUIRY APPROACH: THEORETICAL BACKGROUND AND METHODOLOGY

Inquiry and its roots

Inquiry, as I have worked on it with colleagues*,* involves questioning, investigating, exploring, wondering, seeking out, conjecturing and looking critically at whatever we are inquiring into. That might be

- Inquiry in students' mathematical activity in the classroom;
- Inquiry in teachers' exploration of classroom approaches;
- Inquiry in addressing questions and issues to do with teaching and how it can develop to promote mathematics learning.

Briefly, these can be expressed as *inquiry in mathematics* (A), *inquiry in mathematics teaching*(B), and *inquiry in research into learning and teaching mathematics* (C).

Our research into inquiry activity in the classroom is rooted in a sociocultural perspective on learning and teaching deriving from Vygotskian theory that all learning is socially rooted, that learning precedes development and that mediation with more experienced others can enable learners to develop their potential more effectively (e.g., Vygotsky, 1978). We conceptualised our established practices in school and university as *communities of practice*, drawing on Wenger's (1998) concepts of belonging to a 'community of practice' as requiring *engagement*, *imagination* and *alignment*. We learn through engagement in practice, using imagination to interpret our own roles in practice and aligning with the established norms and expectation of the practice. In our inquiry projects we sought to develop an *inquiry community* in which inquiry is emphasised as "a willingness to wonder, to ask questions, and to seek to understand by collaborating with others in the attempt to make answers to them" (Wells 1999, p. 122). Wells emphasizes the importance of dialogue to the inquiry process in which questioning, exploring, investigating, and researching are key activities or roles of teachers and educators (and ultimately, we hope, students). These activities can be discerned through the analysis of dialogue in interactions within the community. So, developing inquiry as a way of being involves becoming, or taking the role of, an inquirer; becoming a person who questions, explores, investigates and researches within everyday, normal practice. As a community of inquirers we aspire to develop an inquiry way of being, an inquiry identity, in our practice as a whole.

So, whether the practice is learning mathematics in a classroom, being a teacher in a school and designing the teaching of mathematics in classrooms, or being a university educator, working with teachers to promote developments in teaching, inquiry enables us to look critically at our practice while engaging with it. Whatever the practice, in order to engage effectively, one has to fit in, align with the norms and expectations; however, it is possible also to question what we are doing and why. We have proposed a concept of "critical alignment" in which, through inquiry, we might become more knowledgeable about practice and therefore more able to engage in alternative ways of being. We suggested that inquiry would start as a tool for alternative engagement and that, through collaborative interaction in inquiry, we would move towards an inquiry way of being, an inquiry community (Jaworski, 2006, 2008b). A key feature of the inquiry at levels B and C was to design inquiry ways of working with mathematics for students in classrooms to foster inclusion and diversity in mathematical learning and understanding.

A theoretical tool – The Teaching Triad

The *teaching triad* emerged from an ethnographic study of *investigative mathematics teaching* (Jaworski, 1994) of a small number of mathematics teachers. Very briefly, this involved engaging students in open-ended and problem-solving tasks through which curriculum-designated mathematical topics would be approached and students' mathematical thinking and understanding fostered. The study led to identification of general characteristics of investigative teaching and to a theoretical construct, *the teaching triad*, which linked the generalized characteristics to three 'domains' of activity in which teachers had been seen to engage: *management of learning* (ML); *sensitivity to students* (SS) and *mathematical challenge* (MC). This triad attempted to provide a framework to capture essential elements of the complexity of the observed teaching and to generalize these to mathematics teaching more widely. Briefly,

- *Management of learning* describes the teacher's role in the constitution of the classroom learning environment by the teacher and students. It includes classroom groupings; planning of tasks and activity; setting of norms and so on.
- *Sensitivity to students* describes the teacher's knowledge of students and attention to their needs; the ways in which the teacher interacts with individuals and guides group interactions. We acknowledge nature of sensitivity as a) affective c) cognitive s) social and cu) cultural.
- *Mathematical challenge* describes the challenges offered to students to engender mathematical thinking and activity. This includes tasks set, questions posed and emphasis on metacognitive processing.

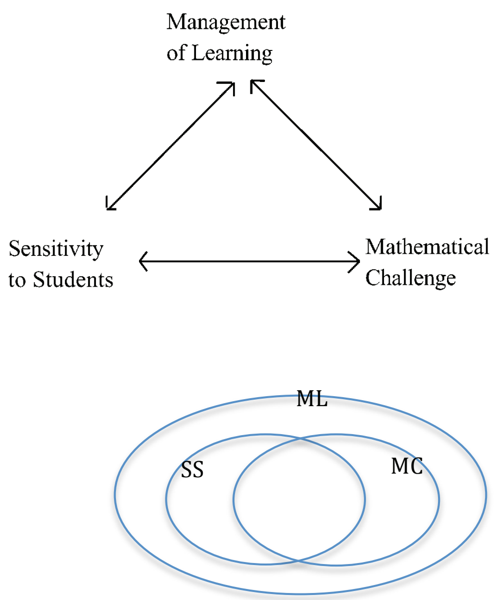

Figure 1: Two representations of the Teaching Triad

These domains are closely interlinked and interdependent as research has shown. The second diagram in Figure 1 represents the relationships between the three domains when the whole of teaching is seen as management of learning. This version was suggested by a teacher in the original project (Jaworski, 1994, p. 144). Further research has shown that the degree of overlap in the second diagram of MC and SS is indicative of where sensitivity and challenge are well related; we see *harmony* in the discourse leading to effective cognitive outcomes (Potari & Jaworski, 2002). The triad has been used particularly to characterize teaching that is 'investigative' in style, or otherwise, carried out with principles of *inquiry*. We believe that the kinds of tasks that can be described as investigative or inquiry-based, as part of a 'pedagogy for all', can enable a way of teaching that can be seen as 'special for all' and promoting of all students' understanding of and enjoyment with mathematics.

INQUIRY AND RESEARCH

I described our collaboration in the TTP as a *community of inquiry*. The teachers were *inquiring* into aspects of their own practice in order to develop practice. The educators were *inquiring* into their use of the triad and whether the triad could be used to analyse developing practice. In a sense, all of us were engaging in research. For the teachers it was a form of action research in their own teaching. They engaged with inquiry in two layers: inquiry in mathematics with students in the classroom; inquiry into teaching as they reflected on what happened when they put their planning into practice and analysed its outcomes. The educators working with these teachers had a dual role: 1) supporting the teachers in their inquiry activity and 2) charting development in the project. In some cases, the first of these raised dilemmas which became a subject of the second. From such events, educators learned about the developmental research process – we might say they engaged in *critical alignment*, as I shall discuss shortly

Central to inquiry activity in the classroom is the creation of opportunity for students to engage with mathematics more effectively – that is to enable better understanding and skill with mathematics. We introduce inquiry in classroom tasks, designed by teachers with teacher educators' support. We believe that engaging in mathematical tasks that are inquiry-based allows multiple directions of inquiry, differing degrees of challenge, mutual engagement and support, harmony in balancing sensitivity and challenge, and response to and respect for difference, all managed carefully by the teacher. Designing such classroom activity has a central developmental focus which involves teachers-as-inquirers exploring the kinds of tasks that engage students and promote mathematical inquiry; ways of organizing the classroom that enable inquiry activity with access for ALL students; and the many issues and tensions that arise related to the classroom, school, parents, educational system, society and politics.

Inquiry in research into learning and teaching mathematics involves teachers and educators *as researchers* undertaking research into

- Responding to the mathematics curriculum
- Task design
- How students respond to tasks
- How the tasks allow sensitivity to and care for different needs
- Learning processes and outcomes

- Issues in social relationships and differing cultures within classroom and school
- Issues in power and responsibility

Clearly teachers are bound to attend to the national curriculum; the ways of doing this within an inquiry process are an important focus of study, as are the design of tasks and their use by students. From a research perspective, it is important also to study the outcomes for students in terms of how we judge learning and compare it with outcomes in the past and those reported in research more widely (for example, in international studies such as TIMMS). These findings/outcomes are part of the third level (C) of inquiry within a project, i.e.

C. Inquiry in addressing questions and issues to do with teaching and how it can develop to promote mathematics learning.

They focus in an overarching way on considerations in levels A and B.

A. Inquiry in students' mathematical activity in the classroom

B. Inquiry in teachers' exploration of classroom approaches

I present now two examples from a project known as the Teaching Triad project, which followed an earlier project, the Mathematics Teacher Enquiry Project (Jaworski, 1998), which developed teachers' own inquiry into their classroom teaching. The TTP involved two of the secondary mathematics teachers from that earlier project together with two teacher-educators. We saw this as a *small community of inquiry* aiming to develop the classroom teaching of mathematics. These (experienced) teachers wanted to use the triad to explore aspects of their own teaching; the teacher-educators agreed to support them and used the triad to analyse data from their classrooms (Potari & Jaworski, 2002). Both teachers had very strong views on what they wished to achieve in their teaching: Jeanette was strong on developing students' self-esteem but felt that her *mathematical challenge* could be strengthened. Sam, an enthusiastic mathematician, knew that for some students his mathematical approach could be *too challenging*, so he wanted to work on being more *sensitive to students*. Both teachers *managed the learning environment* to promote development according to their chosen focus. We observed their lessons, talked with them before and after lessons and held many meetings outside school to discuss progress and issues. The issues raised were not always straightforward or easy to deal with for either teachers or educators but both groups learned from the experience of working together in these ways (Jaworski & Potari, 2009; Potari & Jaworski, 2002).

EXAMPLES FROM THE TEACHING TRIAD PROJECT (TTP)

Example 1: Developing mathematical challenge

Jeanette's teaching is characterized by an emphasis on individual and group work while whole class discussion takes place mainly for sharing ideas from the group work, for reviewing a test or homework, for introducing and clarifying a task and, relatively rarely, for introducing a new concept. She mostly coordinates whole class discussion, rather than leading it. The group work is mainly on a task of investigative and practical nature while the individual work is based on the textbook or on a structured worksheet which describes a new topic and provides a number of problems for the students to consider related to this specific topic. Jeanette's role is mainly that of a facilitator of learning (c.f., Scott-Nelson, 2001), and includes a form of scaffolding (Bruner, 1985) which she describes as 'pushing' or 'pulling' her students (Potari & Jaworski, 2002, p. 363).

In her interactions with the students she typically encourages reflection and negotiation by asking them to explain and justify what they have done, by praising their attempts and encouraging continuation and extension of their work. She provides help by encouraging peer cooperation, by building links between current and previous work, by simplifying the challenge, and by providing emotional support parallel to the cognitive. In some cases, her attempts to help a student result in closed questions leading the student towards an answer, or she provides an answer. However, she conceives of teaching as process oriented rather than as a product oriented activity and aims for the emergence of students' ideas and strategies, and for building their autonomy, self confidence and understanding. A typical students/teacher exchange can be seen in the following extract:

The teacher wanted students to understand the nature of the least surface area for a cuboid of given volume. Her opening activity had focused on the construction of a box to fit 48 cubes each of side 2 cm. Groups of students worked on this task. We focus on an interaction between Jeanette and two boys, Tom and Stewart. The boys had two different organizations of the 48 cubes: Tom with 48 cubes in a line; Stewart with a 2×4×6 (4×8×12 cm) cuboid. They had each drawn nets of their solids to enable them to calculate surface area (respectively 776 cm^2 and 352 cm^2) for a volume of 384 cm^3. The teacher looked, with the boys, at these two cuboids and through her questioning and their responses we gain evidence of Stewart's appreciation of the key mathematical concept (T: teacher, S: Stewart, To: Tom).

Table 1: Transcription and brief TT analysis:

#	Who	Transcript turn	Brief analysis
1	T	Now I want you to think why Stewart's [surface area] is less [less than Tom's, whose shape is very long and thin]	SS: Engaging with where they are at. ML: Tacitly accepting what they have done so far. MC: encouraging him to *think.* (Pedagogy)
2	S	Cause mine's higher and wider and …	His words provide some clues to his thinking?
3	To	It's easier to fit in the trolley. [He refers to a supermarket trolley]	Trolley, another clue? The social setting is packaging for goods in a supermarket.
4	T	It [Stewart's] is easier to fit in the trolley, yes?	SSa,c: accepting their words and encouraging more – the word "yes" is partially a question
5	S	Because mine's got more height and	Since more seems to be required, he adds

		width than Tom's.	to his detail, but still imprecise
6	T	Right, so it will be (many voices) consequently it's been made …	MC; SSc: she wants more, she prompts them
7	S	Shorter [recall that Tom's is very long]	He's responding to the challenge/encouragement
8	T	Shorter, (many voices in the background)	MC; SSc: she prompts again
9	S	More compact	Yet more precise.
10	T	Right. Compact. Good. Right, now, is Stewart's model the most compact model you can come up with, or is there anything better? Well. I don't know. Let's look at James and John's to see if they've done better	SSa,c: accepts his answer and praises him MC, now another challenging question ML/SSc,s: encourages them to take into account what other students have done/achieved, and to look at it critically (pedagogy)
11	many	Some interruption here from other students to whom the teacher responds	ML: keeping her eye on the whole class while working overtly with 2 students (pedaogogy)
12	T	Stewart, you have done really well so far, OK?	SSa,c: Praising him. Valuing his contribution
13	S	Yes	He accepts
14	T	But you need to make sure you are listening in to the others' designs as well	ML/SSc: She emphasizes valuable cognitive activity (pedagogy)

Jeanette was satisfied with her efforts here to offer more mathematical challenge – that is going beyond praise to push students to think and to express their thinking. In this episode she was rewarded by a form of words that persuaded her of Stewart's understanding of the nature of minimal surface area. She was able also to emphasise the value of sharing results and collaborating on ideas, important aspects of her pedagogy. However, towards the end of the sequence of lessons on surface area, she held a whole class session with which she was much less satisfied. She had asked different groups of students to explain their findings to the class. She had a mathematical vision of what she hoped certain groups would contribute. When one student, Peter, was not able to articulate what she was hoping for, and her prompting did not result in Peter's providing the key words, and the lesson was near the end, and students were restless, she briefly explained to the class what she had hoped Peter would say, and wound up the lesson. She said to us afterwards that she would have to revisit these ideas in the next lesson (despite her plans for other work) because it was quite clear that she was over-ambitious in her expectations of Peter, and she had lost the rest of the class (further details can be found in Potari & Jaworski, 2002). We see her a basis for critical alignment as she tackles these issues with time frames which cannot be adjusted.

Example 2: Developing sensitivity to students

Sam was a very experienced mathematics teacher, highly regarded by school and colleagues. He was an enthusiastic mathematician, innovative in his approach to classroom activity and demanding of students in expecting that they would engage with mathematics in thoughtfully creative ways as he did himself. He had joined his current school as head of the mathematics department only 1 month before the TTP research began. The students of the Year 10 (Y10) class to which we refer were designated by the school mathematics department as a "lower set," suggesting that these students were lower achievers than others in their year group. There were just 14 students in this set. We recognize that terminology here is neither socially neutral nor uncontentious and these factors contribute to our analysis. Teachers had to decide, for any student, which level in the final examination (GCSE) was appropriate, and this was based on students' performance in their allocated sets throughout secondary schooling, and setting was influenced by this examination structure. A school expectation prompted by the national curriculum was that students would engage with "homework" outside classroom hours. Sam's approach to teaching was characterized by a combination of whole class teaching and individual or pair work. His main teaching goal was that his students should understand and be involved in doing mathematics and also develop mathematical skills. This applied to students at all levels, although he recognized a specific challenge with the Y10 class.

> I try and get my lessons based on their understanding and I try to make that the focus of the lesson. And if it doesn't work, it's important and therefore I have to do something to make them understand … Somehow, I think it's not so easy with this Y10 to do that, they are not so easy. And also they are put in a bottom set, and having been put in that they are thinking, 'well OK we are not expected, we are not expected to think in this kind of way', and I really want to think that you [the student] can [think], and I think some [students] do [think], you see; my worry is that some of them just turn off.

Analysis of our observations shows that Sam offered help and support to students by encouraging them to reflect on their actions, asking focused questions, encouraging them to make connections with their previous work, inviting them to contribute to whole class discussion, asking for peer communication, and expressing his goals and leading the students toward them. Often, individual help to a student took place as part of the whole class dialogue or was given in a short talk with a student, or a quick hint, while students were working individually or in pairs. What we saw little of was careful listening to students to make sense of their interpretations of the tasks with which they engaged. Sam saw his strength as a teacher being in offering mathematical challenge at appropriate levels. He wanted to judge this more carefully with respect to sensitivity to students' (cognitive and affective) needs. In practice, there were cases where the teacher's objectives differed from the students' needs and were unrealizable by the students so that tensions emerged. He talked of certain students, or groups of students, being "resistant" to his teaching, while others worked "productively." We emphasize that these were the teacher's words, and we use them in this spirit, rather than, for example, our own theorizing of resistance and productivity. Sam's research in the former project had been directed at exploring reasons for what he perceived as students' resistance (Jaworski, 1998). Our analyses, below, treat such tensions as central to a characterization of the social frame in which teaching–learning activity takes place and throw light on what the teacher saw as "resistance."

We focus on one lesson in which Sam wished to introduce statistical ideas relating to 'average'. He had planned the lesson carefully: students would be asked to look up, in a dictionary, definitions of *mode*, *median*, *mean* and *range*. The class would then discuss these definitions and undertake a task related to them to consolidate meanings. The task involved a set of cards with statements each of which should be linked to one definition; examples are:

Card 1: The sum of the numbers divided by the number of numbers.
Card 5: The … of 2, 4, 1, 3, 4, 1, 5 is 4 because the highest number is 5 and the lowest is 1.

The dictionary task was set for homework. At the start of the planned lesson, Sam asked students to take out their homework and be ready to share their definitions. It transpired that 8 of the 14 students had not done their homework. Some indicated that they did not have a dictionary at home. Sam responded that they could have used a dictionary in the library. He was visibly 'unhappy' with the class as evidenced by his words and body language; they had upset his planning for the lesson. He said amongst other statement that they would receive detention according to school rules for not doing homework, and, with insensitivity to the students' home background, "I'm surprised that you don't have a dictionary at home because I think it's really important that you have a dictionary". Students complained loudly at what they saw as the injustice of these statements.

In the event he gave dictionaries (which he had brought to the lesson) to those who had not done the task. The students who had done the homework read out their definition and there was some discussion. Then the teacher explained the cards task and distributed cards. We now focus on two girls, Amy and Sarah who had not done the homework; they have a dictionary and a set of cards. When the teacher approaches them in his circulation of the classroom, it becomes clear that they do not know how to use the dictionary -- they think it is a French dictionary – their experience of using dictionaries has been in French lessons. Sam showed them how to look up words in the dictionary and moved on to talk with other students. His style was to have a quick talk with them and then leave them to work further. In the process of the lesson he returned to Amy and Sarah several times and we saw him moderate his mediation as the lesson progressed. In the beginning it was to clarify what they had to do, such as showing them how to look up words, start the cards task. When he could see that not much was happening, the girls were chatting about other things, he focused in on particular words "what is *mode*? What does it mean?"; when the girls were unable to respond, he gave them an explanation of *mode* and left them to work again on the cards. When again, nothing seemed to be happening with the girls, he spent more time asking them questions and supporting their engagement (for more detail, see Jaworski & Potari, 2009). At one point he praised Amy: "you thought when you did that", and later "right, thinking Amy, that's good. It was clear that Sam wanted the girls not only to engage but to think about what they were doing (in accord with what he had told us earlier). At one point, Sarah said "I can't do that." The teacher showed her a specific set of numbers and asked her "what do you have to do with these?"

referring to the ordering of numbers to get the *median*. Sarah asked, “How can we jump them around. How can we put this one there and that one there?” The teacher asked, “Does it make sense what you said to me?”, and Sarah added “I want to save my brain from working”. We see in these words a deeply cultural situation in which the girls reveal the common state of their school work – engagement that does not require hard thinking, which perhaps in a ‘bottom set’ is what they normally experience. Sam, on the other hand, wants them to think, make sense, and understand the mathematical concepts. Here we gain insight to the nature of the ‘resistance’ that he has experienced. We see here considerable evidence of a large gap between mathematical challenge and sensitivity to students with a corresponding lack of harmony in the lesson.

DISCUSSION AND CONCLUSIONS

Use of the teaching triad in our analyses above reveals characteristics of teaching in the two lessons discussed which are manifestations of more general characteristics of the teaching of these two teachers. The involvement of the teachers in this community of inquiry, their expressed wish to develop their own teaching and the insights they brought to their use of the triad is indicative of their commitment to being a teacher. Both were aware of their areas of strength and weakness. They came to their current teaching with a sincere desire to learn more and make changes. The lengthy discussions in the small inquiry community evidenced their recognition of issues that we have outlined above. Tackling these issues in the longer term is the demand of the developmental process: within the life of a busy and often stressed teacher this is not a simple matter. We supported them in articulating possible changes to practice, but only they could implement and sustain changes.

In both cases teachers were experienced and knowledgeable with designing lessons and preparing tasks of an inquiry-based nature to encourage student engagement and mathematical thinking. This was an important part of their pedagogy, not to be dismissed, but more was needed. Their recognition that their ML required more attention to the relationship between SS and MC was a starting point for development. The lessons reported were each one of many observed and discussed. Each lesson revealed issues and tensions of different kinds related to the SS/MC balance. The various issues provided small details of the sorts of changes that might be needed. For Jeanette, a recognition of ways in which dialogue with students could provide opportunities for increased MC, albeit within her extreme awareness of sensitivity. Her reversion to providing her own explanation when under pressure at the end of a lesson helped her to see that time of day, mood of students and her own planning were factors which influenced her pedagogy, and might need more thought at the planning stage, or consideration in the contingencies of a lesson. However, it was also important to recognize that contingency was often beyond perfect solutions at any one time. For Sam, recognition of his lack of attention to affective, social and cultural aspects of his SS was revealed over and over again, and our emphasis on these factors through the tensions revealed in his lessons was a force for modifying his planning to take more seriously the elements of sensitivity his teaching seemed to lack. For me, this is a clear example of the exercise of critical alignment in seeking to modify his practice related to his learning through inquiry in practice.

I have provided small examples from TT research which highlight the value of the triad for analyzing teaching and helping teachers to think about their pedagogy and modify their practice accordingly. In relation to the broader issues discussed in my introduction, I am aware that the contexts and cultures of these examples only just start to touch on the depth of concern the issues raise more broadly. In other contexts and cultures, different issues will be there. I am not pretending that the triad offers any kind of magical solution for teachers in recognising and dealing with issues. Indeed, the attention here to the use of the triad to reveal issues for attention and development happened within the small community of inquiry in which the teachers had the in-depth attention of teacher educators to encourage and support their development. In other writing (e.g., Jaworski, 2008), I have emphasized the importance of some community of inquiry, whether it includes teacher educators or other teachers or both, is central to the developmental process. I see the triad as having most potential within such an environment. For the educators themselves, there are also issues. Other research has shown that teacher educators, in their work to support teachers in developing teaching, also come up against issues which lead to critical alignment (Goodchild, 2008; Jaworski, 2008). These theoretical ideas are not divorced from practice but entirely central to developmental activity in practice.

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Sub-theme 1

Effective initial and continuing mathematics teacher education

Investigating a preservice secondary school teacher's knowledge of solving quadratic equations

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This qualitative study investigated a Malawian preservice secondary school teacher's subject matter knowledge of equations. The study was carried out with one preservice secondary school teacher. Data were generated using a Subject Matter Knowledge paper and pencil test and a task-based interview. The data were analysed using thematic analysis. Results indicate that the preservice secondary school teacher demonstrated some evidence of knowledge of solving quadratic equations, but he was not able to explain why the methods worked. The implications of these findings for mathematics teacher preparation are discussed.
Key words: Teacher knowledge, preservice, secondary school, quadratic equations

INRODUCTION

Teacher knowledge is important for students' learning. Empirical studies have shown that teacher knowledge influences and affects the quality of teaching and learning (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). Studies of beginning and experienced teachers also reveal that teachers' understanding of and agility with the mathematical content affects the quality of their teaching. Thus, teachers must know the content thoroughly in order to be able to present it clearly, to make the ideas accessible to a wide variety of students, and to engage students in challenging work (Ball, Thames, & Phelps, 2008). Thus, in order to support students' learning of algebra and equation solving, teachers need to know and be able to do more than doing the mathematics for themselves. Despite this call by researchers, Malawian students perform poorly in national examinations (Malawi National Examinations Board, 2008–2013). Analyses of Malawi National Examinations Chief Examiners' Reports for years 2008 to 2013 indicate students' poor performance in Algebra in general and quadratic equations in particular. A number of researchers agree that the concept of equations poses challenges to students. (Makgakga, 2016; Makonye & Nhlanhla, 2014); In addition, research reports on mathematical knowledge for teaching among Malawian preservice secondary school mathematics teachers are less available. Thus the purpose of this study was to investigate a preservice secondary school teacher's knowledge of solving quadratic equations. Focusing on preservice teachers will help improve teacher education so as to produce teachers who are equipped with knowledge and skills to teach Mathematics effectively.The results from this study could thus inform preservice teacher educators about the content of Mathematics teacher preparation.

THEORETICAL FRAMEWORK

Two constructs guide the theoretical framework for this study: Shulman's conception of teacher knowledge and equations. Shulman (1986) developed a framework for transforming subject matter into pedagogy. He divided teacher knowledge into three categories: subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curricular knolwedge (CK). Shulman argues that in teacher development, educators and researchers should focus on both SMK and PCK in order to help preservice teachers to transform from expert students to novice teachers. On SMK, Shulman argues that a teacher should not only understand that something is so, but also why it is so. Knowing 'that' involves knowledge of rules, algorithms, procedures, concepts and principles that are related to specific mathematical topics in the school curriculum. Knowing 'why' includes knowledge which pertains to the underlying meaning and understanding of why things are the way they are (Even & Tirosh, 1995). Knowing 'why' also affects teacher's decisions about the presentation of the subject matter.

The second construct of the theoretical framework for this research, the study of equations, largely draws from the work of Kriegler (2007). Kriegler developed a framework for algebraic thinking which is based on many years of work. Kriegler asserts that algebraic thinking is organised into two major components: the development of mathematical thinking tools and the study of fundamental algebraic ideas. Of these two components, mathematical thinking tools informed this study. Mathematical thinking tools are analytical habits of mind. They are organised around three topics: problem solving skills, representation skills, and quantitative reasoning skills. These thinking tools are essential in many subject areas, including mathematics; and quantitatively literate citizens utilise them on a regular basis in the workplace and as part of daily living (Kriegler, 2007). Thus, to develop and enhance students' algebraic thinking, preservice secondary school teachers themselves need to exhibit high levels of algebraic thinking skills and be able to articulate what it is that they are doing.

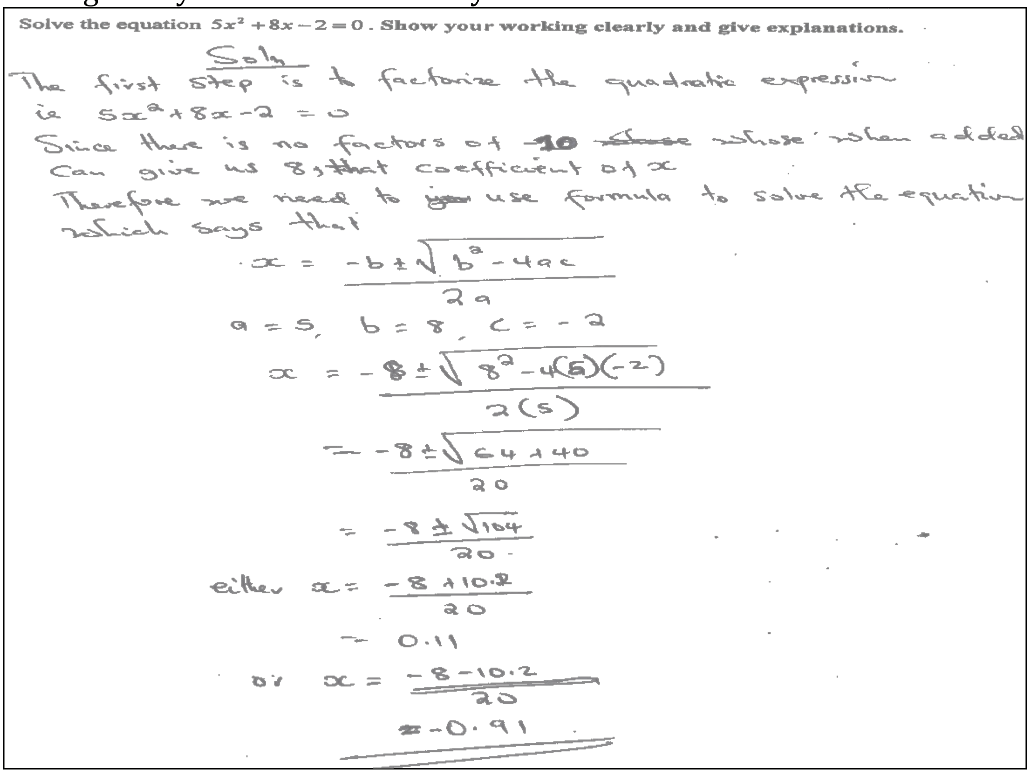
METHODOLOGY

This was a qualitative descriptive case study involving one preservice secondary school mathematics teacher named Mwati (pseudonym). He was a diploma in education student in a three-year programme at a college of education in Malawi. He was trained as aprimary school teacher and had taught in primary school for two years before joining the secondary school teacher education programme. When data for this study were generated, he was in the final year of study. Being one of the best students in his class, he was considered an "information-rich" case for in-depth study (Yin, 2014). A subject Matter knowledge test and a video-recorded semi-structured interview were used to generate the data. The video recorded interview allowed for multiple, in-depth rounds of analysis of the data (Girden & Kabacoff, 2011). Test and interview tasks reported here were adapted from a Malawi secondary school mathematics textbook (Gunsaru & Macrae, 2001) and were piloted before the main study. Data were analysed using thematic analysis (Ritche & Lewis, 2003). Themes were developed *a priori* from the theoretical framework as well as *a posteriori* the data.

FINDINGS AND DISCUSSION

Findings indicate that Mwati was able to solve quadratic equations during the test and interview. He used factor method by difference of two squares, factorisation by trial and error, completing the square, the quadratic formula and by graph. During the test, Mwati solved the equations $25m^2 - 1 = 0$, $5x^2 + 8x - 2 = 0$ and $3x^2 + 7x - 6 = 0$. For instance, to solve the equation $3x^2 + 7x - 6 = 0$ in Figure 1, Mwati firstly factorised the quadratic expression on the left side and saw that factor method did not work. Then he realised that there are no factors of -10 whose sum is $+8$, the coefficient of x and thus, used the quadratic formula. During the interview he solved the equation $x^2 = 2x + 8$ by factor method, quadratic formula, completing squares and by graph in that order. Table 1 illustrates Mwati's exploration of the methods of solving the quadratic equation $x^2 = 2x + 8$.

The findings show that Mwati was able to explore multiple approaches to a problem, to display relationships visually and to translate among different representations. Exploring mathematical problems using multiple approaches gives teachers and students opportunities to develop good problem-solving skills and experience the utility of mathematics (Kriegler, 2007). When solving the equations, Mwati used rules of algebra to solve for the unknown and thus indicating deductive reasoning. Findings also show that Mwati formulated the equations $x^2 - x - 2 = 0$ and $x^2 - 3x - 4 = 0$ from two given graphs respectively indicating ability to reason inductively.


Solve the equation $5x^2 + 8x - 2 = 0$. Show your working clearly and give explanations.

Soln

The first step is to factorize the quadratic expression

ie $5x^2 + 8x - 2 = 0$

Since there is no factors of -10 ~~those~~ whose when added can give us 8, that coefficient of x

Therefore we need to ~~you~~ use formula to solve the equation which says that

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 5, \; b = 8, \; c = -2$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(5)(-2)}}{2(5)}$$

$$= \frac{-8 \pm \sqrt{64 + 40}}{20}$$

$$= \frac{-8 \pm \sqrt{104}}{20}$$

either $x = \frac{-8 + 10.2}{20}$

$= 0.11$

or $x = \frac{-8 - 10.2}{20}$

$= -0.91$

Figure 1: Mwati's solution process to the quadratic equation $3x^2 + 7x - 6 = 0$

Preservice Teacher's Knowledge	Examples from Mwati
Exploring multiple approaches to a problem	**Mwati**: Yes, because this is factorisation, I will also use quadratic formula to solve the equation **Mwati**: I think the problem can also be solved by quadratic formula ... We can also use completing the square.
Displaying relationships visually / Translating among Different representations	**Mwati**: A method I can call the final one is graphical method. **FM**: Can you now solve the equation graphically? **Mwati:** We need to equate this equation to y. Now, it will be $x^2 - 2x - 8 = y$. Now from there we need to find the range of values of x. Maybe, we can start from -3 to 5. This range has just been chosen arbitrarily. Now we need to have a table whereby we are saying the values of x should range from -3 to 5.

Table 1: Mr. Mwati's Knowledge of solving the quadratic equation $x^2 = 2x + 8$ during the interview

Although Mwati displayed some conceptual understanding both during the test and interview; we cannot assume that he has well-articulated subject matter knowledge. He did not indicate any knowledge of why he carried out particular actions in his solution processes. Thus Mwati lacks an important aspect of subject matter knowledge. Being able to use an algorithm to solve a problem and failing to justify steps in the algorithm is an indication of procedural knowledge (Hiebert & Lefevre, 1986). Even and Tirosh (1995) also argue that "knowing that" though certainly important is not enough. "Knowledge which pertains to the underlying meaning and understanding of why things are the way they are, enables better pedagogical decisions" (Even and Tirosh, 995; pp. 9).

CONCLUSION

In this study, Mwati's subject matter knowledge of quadratic equations was investigated through a paper and pencil test and a task-based interview. Findings show that Mwati displayed success when solving the equations using factorisation, quadratic formula, completing the square and by graph. When solving the equations, he was able to explore multiple approaches to a problem, to use rules of algebra to solve the equations and hence displayed deductive reasoning. However, Mwati displayed procedural knowledge such that he was able to solve the given equations, but did not appear to be able to explain why the methods worked. It could thus be expected that Mwati lacks the knowledge necessary for teaching equations effectively to his students. Since this apparent lack of knowledge could be observed with one of the best students, it appears that Malawian teacher education needs to emphasise on the 'why' questions more and go beyond emphasising on the procedural aspects of equation solving. This aspect is lacking in contemporary education of secondary school mathematics teachers in Malawi. With emphasis on 'why' questions, teachers will be able to elicit and interpret students' thinking, identify misconceptions, provide tasks and pose questions that will guide students' interpretations of mathematics. The teachers will also be able to find appropriate strategies for inducing cognitive conflict that will enable students to deconstruct their 'naïve theories' and reconstruct correct mathematical conceptions.

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Promoting quality teaching through a learning study at an initial teacher education institution

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In many universities, academic staff performance is measured against members' contribution to teaching, research and service but there are rarely staff development programmes to help staff improve the quality of their teaching. This paper explores how a learning study impacted shifts in the quality of teaching. The learning study is informed by situated learning in an authentic environment. For one lecturer, a series of lecturers in mathematics were observed and critiqued by peers. With the Reformed Teaching Observation Protocol (RTOP), the rating of the peer reviewed lecturer increased from an average of 2.78 out of 5 in the first observed lecture to an average of 4.28 on the fourth observed lecture. Also the student rating of lecturer performance improved. This occurred as a result of practice shift from subject centred teaching to one that was both subject-centred and student-centred. It was observed that students' participation and learning enjoyment increased as a result of the learning study.

BACKGROUND

Research has shown that the shaky subject matter knowledge of many mathematics teachers is a factor of the mathematics underachievement trap. What role can Initial Teacher Education Institutions play in this scenario? We believe that one of the most important roles of ITEIs is to equip future teachers with not only adequate disciplinary knowledge so that they are qualified to deal with it but also pedagogy so that they know how to teach well what they know. The authors believe that one of the best ways to teach student teachers teach is for teacher educators to be model teachers themselves. To be better mathematics teacher educators we set up a lecturer learning community to improve professional practice.

Professional Teacher Learning Communities

According to Cheng & Mun Ling (2012), a learning study is cyclic with the stages of preteaching conferences, the teaching of one member being observed by peers, and postteaching conferences. The learning study is in the form of Professional Teacher Learning Communities (PTLCs). PTLCs are ongoing processes with the purpose of enhancing professional development (Loughran, 2007; Lawson, Abraham & Renner, 1989). PTLCs were popularised as learning studies and benchmarked from the Japanese (see Cheng & Mun Ling, 2012). PTLCs were popularised as a result of Japan's worldwide dominance in international mathematics and science comparative tests. Significance of the research Peer review of teaching, as well as lecturer evaluation by students is one of the key requirements in teaching quality at our institutions. When the first author approached his faculty on the issue of peer evaluation, he was advised to approach a well qualified and exemplar staff member in the linguistic department, a staff member who had won the University's Faculty of Humanities Teaching and Learning award (this staff member is the second author in this article); and a mathematics subject expert. As the peer evaluation progressed the first author saw the peer review process as an opportunity to research on practice. Besides, the first author also felt that this was an opportune time for him to learn to improve his teaching from his colleagues particularly given that this was a cross disciplinary team. The first author and second author then agreed to take an intentional reflective approach to the whole peer evaluation process with the aim of finding out how such an approach can promote quality teaching among peers in tertiary institutions. Therefore, while the normal peer evaluation was being done, our interest turned to reflect and learn from the process, hence the learning study. In this paper, we report on the insights that came from the reflective approach that we undertook during the peer evaluation process. According to Ball (2000), reflective practice need no special design or conjecture as its primary purpose is to help sharpen teachers' foci when deliberating on what is going on in their work. In this study therefore, we used reflective practice as a tool that offered us a means for examining the whole peer review process.

Research Question

The research question that guided this study was: In what ways does an intentional reflective approach during a peer review process of teaching influence shifts in lesson delivery and teaching practice?

CONCEPTUAL FRAMEWORK

A Learning Study (LS) is an approach which is used by teachers to improve their professional competence on the basis of observations of fellow teachers and students' interviews (Stepanek, Appel, Leong, Mangan, & Mitchell, 2007). A LS has two key features; it is collaborative and it uses action research principles (Cheng, 2009). The collaborative approach is used by the teachers to help them to jointly construct pedagogical content knowledge for the purposes of improving their teaching and ultimately students' performance. The principles of action research that are employed in a LS are the four key stages of planning, implementing, observing and reflecting leading to a new cycle (Kemmis & McTaggart, 1988). However, unlike in action research where the research work can be done by an individual, in LS, the work is a group activity throughout. In LS, what is planned are lessons which are then taught by individual members of the group while the rest are observing and the reflecting is done by the whole group. While in action research the focus is on solving an identified problem, in a LS, teachers establish and work towards long term goals that focus on what they want their students to become and achieve (Stepanek, Appel, Leong, Mangan, & Mitchell, 2007). In this study, the LS approach was used by teacher educators at a Higher Education Institution with the aim of improving teaching and ultimately students' performance. One way to further learning studies is for practitioners to watch good (and bad) video clips of lessons. Berk (2009) points out that 'a video can have a strong effect on your mind and senses... it is so powerful that you may download it off the internet so you can relive the entire experience over and over again' (p.2). This is because video clips help practitioners to draw powerful cognitive and emotional experiences. Video clips tend to cater for multiple intelligences (Gardner, 2000) in that people learn differently through visual, emotion, auditory (musical), spatial and so on. Further videos can be replayed by a viewer to internalize the nuances of the message. Thus video can provide powerful teaching models that members may emulate. I this research video clips teaching were used from the co-author's previous teaching. The coding of observed lessons was based on the Reformed Teaching Observation Protocol (RTOP) adapted from Sawada, Piburn, Falconer, Turley, Benford & Bloom (2000).

DATA COLLECTION PROCEDURE

The study had four participants: the first author who was being peer reviewed and three peer reviewers. Students (n=163) completed questionnaire to evaluate lecturer performance. Also data collected was in the form of notes, audio-recordings of pre- and post-conference meetings, individual and team reflections and observation reports. 163 questionnaires were issued to students to rate the lecturer performance of teaching after the learning study interventions were done. These were analysed by a university computer system. In the questionnaires, they were 25 descriptions of which the students were to evaluate the lecturer; rating him from A: strongly agree, to E: strongly disagree with C being neutral.

Teaching video clips comparison

The teaching video clips were from the second author's lectures taken during the study of her own teaching which won her the 2014 Teaching and Learning excellence award. The video clips showed examples of what she described as mediocre teaching and quality teaching. The reviewer and the reviewee would discuss aspects of the video clips to explain why others were regarded as good teaching and the others were regarded as not so good.

DATA ANALYSIS

Data analysis is based first from the perspective of peers in the learning study and secondly from the students' evaluation of the lecturer's teaching at the end of the cycles of the learning study. Analysis from the perspective of peers We show below some examples of forms of data collected. Data from pre-lecture conference were in the form of audio transcripts, example is shown below.

All the 163 students completed the questionnaires. Students' questionnaire data were analysed with the help of a computer system dedicated to multi-choice assessments in the university. The analysis showed items in which the lecturer did well and did not do so well.

The first post-lecture conference was a turning point in the way the reviewee (first author) approached his teaching. In the first post-lecture conference, the reviewee learnt about aspects of teaching that he had never thought of before, such as the importance of including in one's teaching the relevance of mathematical concepts to be taught. When the reviewee added this aspect of relevance in his next lecture by discussing examples of the application of statistical concepts in everyday life, he saw immediate changes in his class. Students visibly showed interest in the content of the lesson. The reviewee could hear expressions like "Oh", "Okay" coming from students. These observations confirmed what was said by Nyamupangedengu (2015) that students are motivated and show interest when they find personal relevance in the content they are learning. From the post-lecture conference meeting the reviewee also got reminded of the many good teaching habits that he had stopped practicing such as working out maths examples on the chalkboard for and with students. This approach promoted interaction between the reviewee and his students and further increased participation as it provided students with a chance to immediately assess their own understanding of concepts and to seek clarification where they were in doubt. This is a basic aspect of good teaching that the reviewee had abandoned. This confirmed how peer review can promote quality teaching; reminding colleagues of what good teaching entails. After the first post-lecture conference meeting, the reviewee began to focus more on students' interaction with the learning material.

CONCLUSION

In the last review there was evidence of shifts in practice in that both elements of NCTM Principles and Standards (NCTM, 1989, 2000) and elements of RTOP (Sawada et al., 2000) were taken into account as the lecturer began to take into account not only of the curriculum principle of important mathematics but also began to involve students more in the lecture by giving them time for more involved participation. This also exemplified by inviting students to evaluate the lecturer's performance. Thus shift increased students' communication and conversations that enable them to reason more fully and justify their thinking which led to more learning. Such an approach caters for the diversity of students and caters for social justice in that all students' points of view in learning are taken into account. As students were given more worksheets they learnt to solve problems not only during home work and assignments but also in class through the worksheets they went through during the lectures. Problem solving as an important standard of mathematics teaching and learning was catered for. Although the reviewee had taught mathematics for more than two decades in high school and tertiary institutions, it was clear from this short peer review that there was still room for him to improve hi lecture delivery.

This report shows the reviewee's radical shift from being a subject centred practitioner to a learner centred one. The subject centred practitioner is mostly influenced by strong discipline classification and weak framing (Bernstein, 1996). There we are shift to both strong classification and framing, in which subject matter continued to be important but also the role of students in learning also became central. The reviewee was no longer just interested in dishing out mathematical knowledge but also began to engage learners at a higher level. This to a certain extent slowed the pace of the lecture, but this was a small cost to pay given the exuberance of students as they began to engage with statistical concepts. That way, they began to solve statistical problems, communicated their ideas and so learnt from each other. Also since their ideas were in the public space, students were bound to reason more to justify their mathematical ideas. The reviewee noted sharply the importance of respecting students' prior knowledge which might have errors and the relevance of previously learnt material. He learnt to never assume anything easy, everything had to be explained even though it might appear superficial to him. Seeing a spirited engagement of students with each other, or with me about a mathematical task at hand was one of the first author's most insightful and rewarding moment of this learning study.

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Investigating preservice teachers' learning through a pedagogy of enactment

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Research on pedagogies of enactment such as microteaching, lesson study, and rehearsals, have mostly focused on the opportunities that they provide for preservice teacher (PST) learning and what PSTs learn through these pedagogies; little is known about how that learning takes place. This study examines PST learning through implementation of the Bellringer Sequence (BRS) in a secondary mathematics methods course. The BRS is a pedagogy of enactment centered around bellringers—brief mathematical tasks implemented as students arrive for class. This study investigates both what and how PSTs learn through the BRS in the context of a secondary mathematics methods course. Initial findings reveal the complexity of the nature of preservice teachers' learning through the BRS as reflected in PSTs' dual roles of learner and evaluator and multiple foci which include mathematics and pedagogy.

INTRODUCTION

Concerns about the disconnect between what teachers learn in teacher preparation programs and what goes on in actual classrooms have led to calls for a greater emphasis on *practice* in teacher education (Ball & Cohen, 1999; Ball & Forzani, 2009). Approaches to teacher education that focus on learning the work that teachers carry out, without any implications for where that learning takes place, are referred to as *practice-based approaches* (Ball & Cohen, 1999). Pedagogies of enactment are considered to be practice-based approaches to teacher education because they focus on PSTs' learning of what teachers do (Lampert et al., 2010). Pedagogies of enactment that have been clearly defined in the literature are micro-teaching (e.g., Allen, 1966), lesson study (e.g., Lewis & Tsuchida, 1998) and rehearsals (e.g., Lampert et al., 2013). Research on these pedagogies has shown what preservice teachers learn through these pedagogies and explored the opportunities that the pedagogies provide for that learning (e.g., Lampert et al., 2013). However, little is known about how PSTs' learning takes place through these pedagogies. This study examines *what* and *how* PSTs' learn through a pedagogy of enactment—the Bellringer Sequence (BRS)—that draws from the affordances micro-teaching, lesson study, and rehearsal.

LITERATURE REVIEW

Micro-teaching is characterized by reduced complexity of teaching, reflected by the brevity of PST's enactment of a lesson, focusing on the PST's development of a specific skill with a small group of students or peers. Early studies on micro-teaching were predominantly experimental studies that highlighted the effectiveness of micro-teaching (Hargie, 1977), but provided very little understanding of the process of micro-teaching in terms of skills, interactions, and how preservice teachers learn to become teachers. More recent studies (e.g., Sezen-Barrie, Tran, McDonald, & Kelly, 2014), have attempted to get inside the process of micro-teaching by examining the characteristics of the interactions that take place in micro-teaching. Lesson study is a Japanese form of professional development that involves teachers working collaboratively in a cycle of enactment, reflection, and revision of a lesson, often with the goal of improving some identified aspect of instruction (Lewis & Tsuchida, 1998). Since the use of lesson study in teacher preparation is relatively new, most of the extant literature is related to exploration of its potential for PST learning. Rehearsals are centered around *Intructional Activities* (IAs) which act as containers of core practices. The enactment of IAs is intended to support PSTs' development of core practices of teaching. Studies on rehearsal have provided insight into the interactions between teacher educators (TEs) and preservice teachers (e.g., Lampert et al., 2013). The BRS, which is conceptualized in this study, is centered on the bellringer—a brief mathematical task implemented as students arrive for class. The BRS draws feedback and reflection from the microteaching, lesson study and rehearsal. From lesson study and rehearsal the BRS draws collaboration. From microteaching the BRS draws reduced complexity. The BRS involves four phases: preparation, implementation, debriefing, and written reflection. This study seeks to answer the following research question: *What* and *How* do preservice teachers learn through the implementation of the BRS in a methods class?

Theoretical framing

This study examines PST learning by applying three frameworks—a knowledge and practice framework that addresses *what* is learned, a learning theory framework that addresses *how* learning takes place, and a pedagogical framework that provides a *lens for viewing* preservice teacher learning in the context of a pedagogy of enactment. The knowledge and practice framework utilizes these existing constructs: *content knowledge* and *pedagogical content knowledge* (Shulman, 1986); *pedagogical knowledge* (Grossman, 1990); and *high-leverage teaching practices* (TeachingWorks, 2018). The learning theory framework integrates the emergent perspective (Cobb & Yackel, 1996) and the situative view (Putnam & Borko, 2000). The pedagogical framework is based on Grossman et al.'s (2009) pedagogies of practice related to professional education. The emergent perspective integrates interactionism and constructivism (Cobb & Yackel, 1996).

METHODOLOGY

The study was conducted in a secondary school mathematics methods course at a Midwestern university in the US. The participants were 11 traditional PSTs. Data sources for the study included a pre-course survey, audio records of bellringer preparation conversations, video records of the methods class sessions, interviews with the PSTs, and PSTs written reflections. Analysis was conducted at two levels. The first level of analysis examined *what* was learned and the second level of analysis examined *how* that learning took place. The unit of analysis was an *instance* of evidence of PST learning—*each occurrence of an expression or demonstration that they have learned something* (e.g., an idea, concept, skill). The statement or collection of statements that surface the ideas learned and interaction with those ideas make up the *learning prompt* which was the unit of analysis in the second level of analysis. The second level of analysis involved characterizing the statements made and identifying broad themes across the learning prompts related to how the idea that was learned surfaced and was engaged with by the PSTs.

RESULTS

The different foci of the phases of the BRS allowed for rich integrated learning. With regard to content knowledge PSTs learned new ideas, deepened their understanding of the topics, and had their misconceptions addressed. Instances of learning related to Pedagogical Knowledge broadly fell into two categories: effective bellringers and effective teaching. Almost half the instances on effective teaching involved learning related to use of student thinking. Themes directly related to student thinking included creating space for students to think, using students' ideas rather than focusing on one's own ideas, and engaging more students in discussion. Instances of evidence of learning related to Pedagogical Content Knowledge fell broadly into three subcategories: the structure of the bellringer tasks, selecting and sequencing ideas, and making connections across ideas. Instances of learning were expressed for the following High Leverage Practices (TeachingWorks, 2018) *Leading a group discussion*; *Coordinating and adjusting instruction during a lesson*; *Setting long- and short-term goals for students*; *Checking student understanding during and at the conclusion of lessons*; and *Analyzing instruction for the purpose of improving it*. The analysis of learning prompts for the substance of content in relation to the ideas learned revealed three stages in the conversations: *initiation*—the ideas learned surfaced or were made public, *precisification*—the ideas surfaced were made clear, and *equilibration* (Piaget, 1964)—PSTs related the ideas to their knowledge or experiences. The BRS structure allowed PSTs to take on both the role of teacher and that of learner.

DISCUSSION

This study confirms that PSTs' own experiences in elementary and secondary school mathematics classes does not adequately equip PSTs with the kind of mathematical understanding that would enable them to teach for conceptual understanding (Ball, 1990). The BRS provided an opportunity for PSTs to come to the realization of the need to work on getting better at using student thinking. This is important, considering the role of student thinking in supporting mathematics learning and how this realization positions PSTs to capitalize on opportunities to develop this practice. The different foci of the BRS phases allowed for rich learning in an integrated way highlighting how attention to the foci in the different phases of pedagogies of enactment may be leveraged for PST learning, particularly with a view to optimizing the limited instructional time in methods courses. The stages of precisfication and equilibration within the learning prompts embodied a generative process suggesting collaborative negotiation of ideas that requires a repertoire of knowledge for PSTs to draw from, class norms that allow PSTs to freely share their ideas, and instructor guidance of the process to ensure learning of appropriate content. The structure of the BRS supported PST learning joining with other studies (e.g., Sims & Walsh, 2009) in suggesting that the way pedagogies of enactment are structured has an impact on PST learning.

IMPLICATIONS FOR TEACHER PREPARATION

The potential of pedagogies of enactment in supporting PST learning lies in the way they are structured for use in teacher preparation programs and in their specificity with regard to instructional goals for PSTs. To capitalize on the limited instructional time in methods courses, pedagogies of enactment need to be structured with varied foci to allow for more integrated PST learning. This study also highlighted the important role of reflection in PST learning. Microteaching, lesson study, and rehearsal all have reflection as a feature. However, being deliberate in structuring opportunities for reflection as was done in the BRS with the debrief, 24-hour reflection, and the bellringer reflection paper would provide better support for PST learning. The important role of precisification in PST learning highlighted in this study suggests that there is need for TEs to develop skills that would enable them guide conversations in which important ideas emerge, in ways that would support PST learning of those ideas. One such skill would be to recognize an important idea the moment it surfaces in conversation, even in its unclear form, so that they can guide conversation toward making it precise. Further studies are needed to unpack what it would take for TEs to recognize these important ideas for the different types of learning and how to guide conversation successfully to the precise idea.

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Analysing affordances of a mathematics textbook: implications for teachers' pedagogical design capacity

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A mathematics textbook is without doubt the most popular curricular resource. In the developing world it is the most accessible resource and probably the only available resource for teaching and learning mathematics. However, while research on this most valuable resource is gaining momentum with other fields of research, in Africa and most developing world it is still quite scarce. The present paper derives from a larger study that investigated the relationship between the affordances of the prescribed textbook and the teachers' mobilisation of these affordances (Leshota, 2015). The paper reports on the analysis of the affordances of the textbook to the teacher's practice. The results indicate to a particular configuration and ordering of content areas; and a particular instructional approach.

INTRODUCTION

Analysing the affordances of the textbook was the first phase of a larger study mentioned above. The analysis was conducted in order to understand better what it was that the textbooks were offering the teachers in their work. My experiences of visiting the teachers' classrooms made me ask questions about how teachers use their textbooks, and why they do what they do with their textbooks. As in the whole world, in South Africa, the textbook is highly regarded as a resource for teachers as well as learners (Chisholm, 2013). Moreover, with its apartheid past and the low quality of education for the majority of mathematics teachers, the textbook serves as an 'educative' (Davis & Krajcik, 2005) resource for these teachers. However, textbook research in South Africa is quite minimal, hence why this study is necessary.

While textbook analysis and comparison accounts for more than 60% of all textbook research worldwide (Fan, Zhu, & Miao, 2013), the angle in that research is mainly on comparison of textbooks from different countries and not the analysis of the affordances of the textbook to the teacher's practice as proposed in this paper. The work by Valverde et al. (2002) is a good beginning of dissecting the mathematics and science textbooks used in the TIMMS study into its affordances, but as far as I am aware, there are no other studies that actually determine the affordances of the textbook.

Affordances are found in research where they are used more colloquially than intended when they were invented. The next section provides the perspective from which I consider the notion of affordances.

AFFORDANCES

Gibson (1977) invented the term *affordances* of the environment and defined them as "what it [environment] *offers* the animal, what it *provides* or *furnishes,* either for good or ill"(p.). More than the definition itself, the importance of this construct to the present paper lies in the reasons for inventing the term. As he points out

> The verb *to afford* is found in the dictionary, but the noun *affordance* is not. I have made it up. I mean by it something that refers to both the environment and the animal in a way that no existing term does. It implies the complementarity of the animal and the environment (p. 127)

This is about the complementarity of the teacher and the textbook. The question for this paper is what the textbook *offers*, *provides* or *furnishes* the teacher's practice; and if affordances exist independent of the teacher? Gibson (2015) is quite explicit that "the affordance of something does *not change* as the need of the observer changes. The observer may or may not perceive or attend to the affordance, according to his needs, but the affordance, being invariant, is always there to be perceived" (p. 130).

What this highlights is that the analysis of textbook affordances and of how the teachers perceive them are independent processes which need not be carried out together at all times. This hence allows for the analysis and reporting of the affordances of the textbook independent of whether the teacher perceives them or not in this paper.

SITUATING THE STUDY IN LITERATURE AND THEORY

The study is situated in sociocultural theory (Vygotsky, 1978) and the use of tools as mediating artefacts between the subject and the object (Vygotsky, 1978; Wertsch, 1991). The study aligns with the perspective of 'use' that is relational, between the teacher and the textbook. Their collaboration in a dynamic interrelationship, allows for each to shape each other, and for both to shape the outcome of instruction (Remillard, 2005; Stein & Kim, 2009). The collaboration between the teacher and textbook is influenced by the respective features that they bring to their interaction.

Among all of the features the textbook brings into the interaction with the teacher, the study focusses only on the *structure* which Remillard (2012) describes as "the nature and organization of the content of the curriculum, the particular mathematical concepts and goals, and the underlying pedagogical assumptions" (p. 110). Valverde et al. (2002) define the structure with how the *presentation formats* and *performance expectations* of learners are incorporated into its pedagogic structures. And, Ensor et al (2002) distinguishes between two dominant pedagogic approaches of mathematics textbooks, the *deductive* and *inductive* approaches, respectively. In the *deductive approach* "the teacher (or textbook) initially states appropriate definitions or concepts which are then exemplified and followed by exercises for students to practice" (p. 22). For the *inductive approach*, a topic is introduced by engaging student in a range of activities students are to master. These activities would lead to definitions and then practice exercises.

The studies above provide this paper with analytical tools for the affordances of the textbook.

METHODOLOGY

Sources of data

Two editions of a textbook series were used for analysis of the affordances of the textbook. During the time of data collection for the teachers participating in the larger study the national curriculum statement was undergoing changes, which meant that the textbooks were being realigned with the new curriculum statement. The three schools from which the seven participating teachers came from, were using the same prescribed textbook, but also had access to the new edition. The topic chosen for the study was Grade 10 Functions, and with respect to the changing curriculum, there were no changes to the curriculum around Grade 10 Functions.

Analysis

The process of analysis was informed by the elements of structure as provided by literature, that is, i) the *nature of the content* about the content areas to be taught as well as the ordering of the content areas in the textbooks; ii) partitioning blocks of content as *presentation formats*, which include representations such as definitions, worked examples, practice exercises, and so forth; and, iii) the *performance expectations* for learners, actions on tasks expected of learners (Leinhardt et al., 1990). However, the performance expectations will not be part of the analysis reported in this paper.

The analysis for the nature of content focused on the major content areas and their sequencing. For the presentation formats, each page of the textbook was analysed for the different partitioning blocks which were coded. The sequencing of the different presentation formats was also recorded. The results on the *nature of content* and the *presentation formats* are presented in the next section.

RESULTS

Nature of Content

The *nature of content* for both textbooks yielded four content areas in the following order : 1) Notation and terminology, dealing with introductory issues of function notation and terminology; 2) Properties of functions, in which properties of three function classes, namely, parabola, hyperbola and exponential functions were determined; 3) Transformations of functions of the form, $y = af(x) + q$, including linear functions, and; 4) Interpretation of functions, which includes sketching graphs, determining equations of graphs and interpreting sketch graphs. Thus, with respect to the content, the textbook affords the teacher's practice with the content to be taught and learned, as well as specific ordering of the areas.

Presentation Formats and their Sequencing

From both textbooks, five distinct presentation formats emerged as: the *Explanatory Text* (definitions, explanations, narratives); *Worked Examples*; *Practice Exercises*;

Activity; and *Assessment Exercises*. The presentation formats in themselves do not point to the embedded approach of the textbooks; it is their sequencing that shows whether the approach adopted by the textbook would be inductive or deductive. In both textbooks, the sequencing showed two distinct patterns: it either began with an *explanatory text* or *worked examples*, thus exhibiting a *quasi-deductive approach* (Leshota, 2015); not quite deductive according to but close. On the other hand the sequence would begin with an *activity*, that is, a *quasi-inductive approach* (Leshota, 2015). Table 1 shows how the sequencing looked like in both textbooks for the different content areas

Content	Textbook	Sequence start	Approach
Notation and terminology	old new	worked examples explanatory text	quasi-deductive quasi-deductive
Properties of functions	old new	activity activity	quasi-inductive quasi-inductive
Transformations of functions	old new	activity activity	quasi-inductive quasi-inductive
Interpretation of functions	old new	worked examples worked examples	quasi-deductive quasi-deductive

Table 1: Sequencing of Presentation Formats in the textbooks

Table 1 shows that both editions of the textbook exhibited a similar approach to the teaching of functions. For the content area where learners needed to form generalisations of properties, the quasi-inductive approach that is more investigative was adopted; and the didactic, quasi-deductive approach was adopted where procedures needed to be exemplified.

CONCLUSION

The analysis of the affordances of the textbook shows that the textbook offers particular ordering of content areas, and a particular approach to the teaching of functions at grade 10. A quasi-deductive approach is adopted where notation and terminology are involved as well as where procedures need to be exemplified. It offers a quasi-inductive approach when determining properties of functions and their transformations. These affordances of the textbook in turn need to be perceived by the teacher in order to maximise effective utilisation of the textbook. However, as Ensor et al. (2002) and Leshota (2015), pointed out, most teachers preferred the deductive approach to teaching, implying a 'miscommunication' between the teacher and the textbook. This has implications for the outcome of instruction.

The paper recommends a direct focus for identifying textbook affordances for teachers at pre-service as well as in professional development programmes.

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Comparative Study: The case of mathematics Teacher Preparation (MTP) programs in Norway and Ethiopia

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This abstract presents a comparative study of mathematics teacher preparation (MTP) in the Ethiopian and Norwegian education systems. How mathematics teachers are trained in these countries is compared by considering the programs, structures, syllabi and content knowledge for teaching mathematics. Document and syllabi analysis method with interview of mathematics teacher educators is applied together with different conceptual and analytical frameworks to do the comparison. The rationale of the study is to give insight to the extent it contributes to existing mathematics teacher education in Ethiopia to improve the quality of mathematics teaching, by taking the progress made in Norway. The study finding revealed a difference, in general, in the philosophy and learning theories applied in mathematics education among the educators in the countries. In Ethiopia, there is much focus on the teaching of higher-level mathematics concepts, which have less to do with the level of the mathematics being taught at the primary and lower secondary schools. In Norway, there is less focus on the higher-level mathematics content and more emphasis on the didactics or special knowledge to teach the mathematical concepts.

INTRODUCTION

Artigue and Winsløw (2010) argue that comparative studies are useful in investigating the similarities and differences of one construct in two different contexts. The construct under consideration is the mathematics teacher preparation (MTP) programs in the two countries (contexts) as in Tatto (2008, 2014) and Burghes (2011). Cai et al. (2016) have mentioned several objectives of comparative studies, and the rationale to do a binary comparative study in MTP in Norway and Ethiopia lies in line with these objectives. Especially, the study intends to contribute to the quality of mathematics education in Ethiopia by taking the progress made in Norway in the light of the international mathematics education community. In addition, the study can contribute in laying down grounds for further studies and research cooperation, for example, for the

NORHED-project that has already begun in 2017 with the help of the Norwegian Agency for Development Cooperation (NORAD)[3].

In Norway, there has somehow been a unified reform and development of mathematics education since the mid-1970's after several years of rift between modernizers and traditionalists. This could be due to the reform called "the new math", which was gaining ground in the country, mainly influenced by developments in the US and other Western European countries, which were based on modern mathematics (Breiteig et al., 2010). This, in turn, has affected the school curriculum and the program for mathematics teacher education in the country. On the other hand, in Ethiopia, if such a movement has happened or is happening is not known so far. Of course, improving the quality of education is the focus of the Ministry of Education of Ethiopia (MOE, 2010), and one of the strategies implemented to gain quality has been through teacher development programs (TDP), that is, through an improved pre-service teacher education system and an expanded system of in-service TDP (MOE, 2008, p. 6).

This study aims to investigate the in-service MTP programs in the two countries by trying to address the following questions: *Q1: What are the underlying philosophies and learning theories adhered to in the MTP programs? Q2: What are the major differences in structure and organization of the primary and lower secondary MTP programs in these two countries? Q3: What are the similarities and differences between the two programs when it comes to* Content *Knowledge (general pedagogical, pedagogical content knowledge [PCK], subject matter knowledge [SMK]), (Ball, 2008)? Q4: What is the emphasis? Is much emphasis on school level mathematics or advanced mathematics? Is it theory-loaded, or is it practice-based too?* The methods used to address these questions are presented first. Then the data and the comparative data analysis is provided. After that comes the findings, the conclusion and the future work suggestions.

METHOD

Looking for insight into the first question, four people (three mathematics teacher educators and one from the national center for mathematics education) are interviewed in Norway. Whereas, in Ethiopia six mathematics educators from different universities and colleges and one person at the MOE who works at the TDP are interviewed. The conceptual frameworks for the interviews are derived from the literature in Ernest (2014) and Lerman (2014). Questions 2–4 are mostly addressed by analyzing national documents and teacher education programs as well as structures, syllabi and courses in the different MTP programs for primary, lower and upper secondary levels. The teacher educator institutions considered in Norway are from middle Norway, and there could be some differences among institutions in the country. Overview of the programs, including the overall structure of the programme, the entry route, and length of the study, entry qualifications, and course components are described first as in Burghes (2011) and Canadas (2013). The different domains of content knowledge for teaching are identified in the programs, like Subject Matter

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[3] https://www.norad.no/en/front/funding/norhed/news/

Knowledge (SMK), General Pedagogy (Gen.Ped), Pedagogical Content Knowledge (how to teach the subjects, PCK), Practice (Prac.), and Common Course (Com.course), and analyzed if these are presented interwoven together or separate (Ball 200; Schmidt, 2011). The document and syllabi analysis also includes if the courses are loaded with school or advanced mathematics as well as if there is a balance between practice and theory in the training (Wonsløw, 2007; Sorto, 2009).

COMPARATIVE DATA ANALYSIS

Philosophies and learning theories adhered to in the MTP programs

The interviews in Norway are dominated by phrases like *mathematics is fallible and is being created continuously*. *It is science about patterns and relationships; it has something to do with exploring of patterns and relationships and also about reasoning and proof; to see patterns, recognize patterns, and work with patterns. It is a way of thinking; it is something you develop; it is not that fixed; even though you did some things that seem fixed it is more of dynamics.* These statements indicate that the educators' view could be coherent with *the fallibalist* or constructivist philosophy of mathematics education (Ernest, 2014), whereas the response from the Ethiopian mathematics educators was different. Some even do not know which philosophy they follow. But most are in the category that Ernest calls *the absolutis*t viewpoint. The educators in Norway prefer the socio-cultural and constructivist learning theory. On the other hand, the behaviorist learning theory seems to dominate the Ethiopian mathematics educators although there are some inclinations to the constructivist/cognitive learning theory (Lerman, 2014).

Structure, organization entry requirements and duration of the primary and lower secondary MTP programs

Four different types of MTP programs in Norway are presented in this article: the initial teacher training programs for primary (1–7) grades, lower secondary (5–10) grades, and upper secondary (8–13) grades, as well as the graduate teacher training program for mixed levels (lower secondary/upper secondary). In the primary school, teachers are prepared to teach different subjects in addition to mathematics (up to four subjects). While in the lower secondary school, teachers teach fewer subjects. In Ethiopia, there are two main pre-service MTP programs. The first is the primary MTP programs at the college of teacher education (CTE) for teachers of 1–8 grades. Preparation for teaching at 1–4 grades is a self-contained one; it means a single teacher teaches all the subjects. While those being trained to teach in grades 5–8 have to select one out of two streams—generalist or specialist. The generalist stream prepares teachers to teach mathematics and environmental science subjects in the lower primary grades. Whereas the specialist stream prepares teachers who specialize to teach one subject, i.e. languages (mother tongue or Amharic or English) or Art/Music. Recently, in Norway, the entry route to the MTP programs for primary and lower secondary schools has been upgraded to a master's degree. The entrance requirement for these programs are general study competence, at least 35 school points, an average of the minimum grade 4 in mathematics (224 hours) and 3 in

Norwegian (393 hours)[4]. Whereas, entry qualification to the three-year diploma MTP for both lower (1–4) and upper (5–8) grades is graduating from 10th grade with a minimum of 2.00 and above in Ethiopian General Secondary Education Corticated Examination (EGSECE).

Content Knowledge of the programs

Zooming into the five-year master's study of the primary (1–7) grades MTP program, somehow similar to the 5–10 grades, the course content distribution is composed of 4 SMK (Mathematics, Norwegian, and two other subjects) 225 study points (stp), pedagogy 75 stp and practice 115 stp. That is, 20–25% SMK interwoven with the PCK in mathematics, 20% pedagogy and 80 days practice in 6 semesters including 5 days with school overtaking responsibility and 30 more days in the other three semesters. While the 5-year "Lektor Program" for teacher 8–13 grades is composed of 150/142.5 stp mathematics, 37.5/45 stp physics, 22.5 stp PCK, 30 stp pedagogy, 15 stp common courses and 14 weeks (70 days) practice, i.e., about 60% mathematics, 12% pedagogy and 9% PCK. The post-graduate diploma training (PGDT) for grades 8–13 teachers is composed of 15 stp mathematics didactics, 15 stp natural science didactics and 30 stp pedagogy in a year or two program. The present Curriculum Framework for Primary Pre-service MTP program in Ethiopia has PCK emphasis also (MOE, 2012, p.2). The course content distribution in the lower primary MTP program in Ethiopia is as follows: SMK 48 credit hour (cr.hr.) of which 24 cr.hrs is mathematics, pedagogy 32 cr.hr, PCK 11 cr.hrs, 10 cr.hrs practice and common courses 4 cr.hrs common course of the total 105 cr.hrs., i.e., about 46% SMK (23% mathematics), 30% pedagogy and 10% PCK.

FINDING AND CONCLUSION

"Integrated" Content Knowledge

One strong observation we would like to make in this work is the "integratedness" of the SMK general pedagogy and the PCK, if not even the practice, in the MTP in Norway. To explain what we mean by the term "integrated", we analyzed a syllabus at a course level, under the MTP program for grades 1–7 in Norway (in 2016), as an example. One gets the topic "Developing meanings for addition and subtraction." It is presented with the incorporation of a research-based characterization of addition and subtraction problems based on their semantic structure in a context-based problem, which is a vital tool to help children construct meanings to operations. According to Carpenter et al. (2014), there are four[5] main types of contextual word problems: *change, combine, compare and part-part whole*, each with subclasses of addition and subtraction. In this case, the SMK, PCK and Gen.Ped for teaching of the concept of addition and subtraction are integrated. The pre-service teacher-students are expected to practice them during school visit and work together

[4] https://www.ntnu.no/studier/mglu17 or https://www.ntnu.no/studier/mglu510

[5] These four different structures in developing the concept of these basic operations are the result of research called cognitively guided instruction (CGI) by Carpenter et al. (2014).

with pupils in 1st or 2nd grade; this demands a greater professional competency on the side of teacher-students also. By contrast, the knowledge domains in Ethiopian MTP programs are not so integrated.

Content at Higher vs Lower Levels

There is a constant debate among mathematics educators on how much higher content mathematics pre-service teachers should take to be qualified to teach at the primary and lower secondary school levels. In the Ethiopian national document mentioned above, the difference in the training of the lower and upper MTP programs is stated as the content level. And it is explicitly stated that, for teaching at the upper primary level (5–6 grades), one has to have the mastery of the level not only 8–10 but also mastery for the preparatory school level, that is, grades 11–12. This means they should study courses like calculus and the likes. The lower primary MTP program has a course called “Introduction to Calculus”, whereas the upper primary (5–6 grades) MTP program has one more course in calculus with 4 cr. hrs., called Calculus I, Math 362. It is debatable whether spending time to learn the rules and applications of differentiation and integration is beneficial for teaching the basic concepts like number sense, multiplication, division and fractions at the upper primary level. Of course, understanding some concepts about derivation and integration can be helpful, but does that justify taking 4 cr. hrs. course on the topic which has little relevance for the profession teaching at the level of the work? MTP programs are supposed to produce effective teachers as engineering schools produce effective engineers to do that particular job they are trained to do. For that there should be much emphasis on how to do that work at the level (Sorto et al., 2009).

One other crucial observation, which is not peculiar to mathematics but to all the teacher training programs in Norway, is the overlap phenomenon, which is different from what obtains in Ethiopia. There are two phases where the overlap is happening. Overlap-phase 1 is between the primary school (1–7) and the lower secondary school (5–10), and the overlap-phase 2 is between the lower secondary school (5–10) and the upper secondary school (8–13). One of the advantages of designing MTP in such a way is that teachers will be able to assist students in their progress from each stage of education to the next.

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Sub-theme 2

Inclusion And Equity In Mathematics Education (Gender, Multilingualism, Special Needs)

Understanding the choice and use of examples in teacher education multilingual mathematics classrooms

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Examples that teachers choose and use are fundamental to what mathematics is taught and learned and what opportunities for learning are created. Research has shown that the type of examples that are used by the teacher in teaching mathematical concepts can either constrain or enable learners' access to mathematical knowledge (Goldenberg & Mason, 2008). Anecdotal evidence exists that novice teachers struggle to develop a good example space when they teach mathematics. In addition, the fact that most classes in South Africa are multilingual in nature presupposes that examples chosen in multilingual classrooms, how these examples are worded, and how language is used, play an important role in how multilingual learners learn mathematics.

In the present study, I bring together three frameworks which have been used separately by researchers. The emergent framework consists of a three-pronged approach to understanding the exemplifying practices within teacher education. It consists of an amalgam of variation theory (Marton & Booth, 1997), Mortimer and Scott's (2003) notion of meaning making as a dialogic process, and the notion of interacting identities within teacher education (Essien, 2014). I argue that while variation theory provides perspective into the choice of examples by the teacher educator, Mortimer and Scott's framework provides a tool for how language is used to engage with these examples in practice, and finally the framework on interacting identities within teacher education provides perspective on how through the teacher educator's use of language, the different interacting identities in teacher education are constructed/co-constructed. In my study, I show how these three frameworks work together in examining the choice and use of examples in mathematics teacher education classrooms. Preliminary findings on the use of the new (amalgamated) framework show that it can be a tool for understanding what a "good" (use of) example in multilingual pre-service teacher education mathematics classrooms should look like.

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Teaching and learning through the use of an instructional mathematics application programme in multilingual mathematics classrooms

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Mathematics classrooms are often characterized by various teaching aids including, more recently, handheld devices that are often loaded with Mathematics Application (App) so as to provide assistance in enhancing learners' Mathematical understanding. However the very same Mathematics App that is meant to aid the learner into more understanding can be a hindrance if the Language of Learning and Teaching (LoLT) is not carefully considered in the App design stage. This study aims at identifying what language intricacies might exist that currently could be overlooked by focusing on one Mathematics Application called onebillion©.

INTRODUCTION

Studies have shown that early mathematics learning and reading skills are a great predictor of later achievement in the learner's academic life (Duncan et.al, 2007). The Curriculum and Assessment Statement (CAPS) (2011) policy document in South Africa emphasises that "Foundation Phase Mathematics forges the link between the child's pre-school life and life outside school on the one hand, and the abstract Mathematics of the later grades on the other hand. In the early grades children should be exposed to mathematical experiences that give them many opportunities "to do, talk and record" their mathematical thinking". One of the key ingredients to successful mathematics learning is communication within the mathematics classroom. Teachers, the textbooks they use, and the mathematical programmes they employ in the course of teaching, all need to communicate mathematical ideas to learners and learners need to understand and communicate back their understanding. There are many ways that this communication can take place, however one integral part which is essential for successful communication, is language. The connection between mathematics and language cannot be ignored as mathematics is taught in and through language, and especially so in the context of South Africa where multilingualism is the order of the day (Barwell, 2009; Boulet, 2007; Pimm, 1981)

Barwell (2009) argues that the learner's proficiency, or lack thereof, in the language of learning and teaching (LoLT) plays a major role in their mathematics performance, compared to their peers who are monolingual. This emphasises the role that language plays in the everyday teaching and learning of Mathematics. The current mathematics classroom is characterised by the use of various teaching aids to help learners understand certain mathematical concepts and develop a deeper conceptual understanding (Jenkins, 1957). At times, technological applications are used to reinforce mathematical concepts and aid understanding (Ferrini-Mundy & Breaux, 2008)

For the purpose of this research, I intend to investigate how the use of a Mathematics App called onebillion© enables the teaching and learning of mathematics in classroom situations. One way in which this has been achieved is by the home language provision that is offered by the App in order to enhance mathematics access to the learners in a certain social context who are not English first language speakers. Roblyer and Doering (2013) argue that technology can enable the teacher to move towards a more learner centered approach and thus allow for context to be embraced in the mathematics classroom. Furthermore they argue that technology can enable the learner to see mathematics in a less abstract way and rather see and experience mathematics in a more concrete manner and this is most applicable in the elementary school level. Technology affords what they term virtual manipulatives which can be manipulated as need be.

South Africa is one context which has historically been affected by the plague of Apartheid, with effects that are still being felt by the country up to this day (Phakeng & Essien, 2016). In recognition of the necessity to ensure equal access to education for all, the language in education policy (1996) has made provision for learners to receive instruction in their mother tongue supporting conceptual growth as well as ensuring that there is a continuity between the leaners home language and the language of learning and teaching (LoLT). Despite this allowance there is a prominent trend in which the first three years (Grade 1 – 3) learners learn and are taught in their home/first language, however in Grade 4 the LoLT changes into English (Manyike, 2013).

In this study, the mathematics App that I will base my study upon is one which has been developed by onebillion©. This App focuses on core mathematics concepts for the first 4 years of schooling (Grade R to 3 in App store and Google play in a variety of languages (onebillion©, 2018)). The App was piloted in a Province in South Africa and is being used by Grade 1 learners. The App was originally developed in English language and has now been translated into African languages (isiZulu in the case of the current study).

Translating the mathematics register from one language to another is not a straightforward enterprise especially when one language (English) has a long tradition of being used in mathematics (and so has well developed mathematics register) while the other language (isiZulu) is still at a developing stage in terms of the mathematics register. The extent to which language issues can be found in Mathematics Applications that have been translated from a developed language to a developing language has not been an explicit focus in research. This study sets out to investigate this phenomenon. To achieve this, I will focus on the onebillion© Mathematics App that is offered in isiZulu centering on the Mathematical language/register used in the App. This study is therefore informed by the research questions below:

1. How does the isiZulu mathematics language in the onebillion© App compare to the language found in the Curriculum used by the teachers and the learners in Grade 1?
2. What is the teachers' perception of the isiZulu as it is used by the App?
3. What has the App enabled the teachers and learners to understand better? What language issues are imbedded in the use of the App?

Together these questions will help me map the mathematics App to the Curriculum and highlight the teachers' view of the language as used in the App which in turn could have implications on the mathematical understanding of the learner.

The contribution this research will make is that, it is important that we do not overlook the role which language can play in ensuring that the learning that is offered to the learner is not impeded simply by overlooking the importance of language during the design stage of the App.

THEORETICAL FRAMEWORK

The theoretical framework that will be used to inform this study will be that of Engelström (1999) Expansive Activity Model. This theory has its etymology in Vygotsky's (1978) work on mediation, and the work of Leontev on action and activity highlighting division of labour. The main idea behind the expansive activity model is that when analysing how children come to know, it is important to look at the entire system that informs the child's actions, it is important to consider the context in which learning takes place. The child does not come to know on his/her own but in the context of the society in which s/he is part of (Engelstrom, 1999).This theory also allows for the notion of mediation to be taken into account in which case this current study's mediating activity would be that of language (and technology) (Bakhurst, 2009; Martin and Peim, 2009).

METHODOLOGY

A Case Study research approach will be adopted for this research as it will allow for in depth description and understanding of the language nuances associated with the App (Yin, 2016) from the teachers and learners' experiences.

Semi-structured interviews and classroom observations will be the preferred data collection method (Yin, 2016). Two Grade 1 teachers and seven randomly selected Grade 1 learners will be my sample. Furthermore, I would like to review the documents that are used in the traditional mathematics classroom (the teacher and learner materials) as well as the App itself. From this analysis I hope that it will be clear as to whether there is any inconsistencies between the mathematics register that the learners encounter in the mathematics classroom and that encountered in the App (Basit, 2010).

At the time of the presentation, I would have collected data and started with data analysis. It is therefore my hope that I will be able to present preliminary findings from my study.

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Exploring mathematical meaning in two languages and the dilemmas it presents for trilingual mathematics students

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This paper discusses how trilingual undergraduate students of mathematics in Kenya explored mathematical meaning using languages at their disposal as resources. The languages were Language of Learning and Teaching (LoLT) and their home languages. They were however, faced with a dilemma which was either language related or not. The discussion draws on the notion of "language of teaching dilemmas". The data was drawn from a wider study which explored language practices of trilingual undergraduate students of mathematics. The focus was on first year students undertaking mathematics in their programs. Findings show that, while the students viewed code switching as a practice and opportunity that facilitated exploration of mathematical meaning, time was wasted and there was the potential of misinterpreting tasks. In contrast, when a task is not understood in the LoLT, one wastes time and would better switch to home language at the earliest opportunity. An attempt to deal with the dilemmas presented a complex situation. The findings contribute to the field of mathematics education in trilingual contexts and further research is suggested. .

INTRODUCTION

In most parts of the world and particularly in Africa, teaching and learning takes place in a language that is not the students' home language. However, in the process of learning, students may use other languages in their repertoire as resources to explore meaning of mathematical tasks and in fact, in mathematical language and more particularly in mathematical discourse. The exploration is not necessarily a straight forward matter and students may experience dilemmas. In the growing field of mathematics education in context of language diversity, there is need to explore the possible dilemmas and how students deal with them.

A dilemma of code switching among trilingual mathematics students in Kenya is here discussed. In particular, students' exploration of mathematical meaning, and how and why they faced a dilemma of code switching. While previously, the notion of dilemma of code switching has been discussed in teaching contexts, the purpose of this paper is to explore the language and non-language dilemma of code switching among undergraduate students as they individually engaged with mathematics tasks. In particular, the tension between acquisition of mathematical meaning and the challenges it presents.

The paper, therefore, responds to the questions;

1. Why do undergraduate students of mathematics switch to translate mathematics tasks?
2. When do undergraduate students of mathematics switch to translate mathematics tasks?
3. What are the language and non-language dilemmas the students face when they code switch?
4. How do they deal with the dilemmas?

EXPLORING MATHEMATICAL MEANING AND THE LANGUAGE DILEMMA IN TEACHING AND LEARNING PROCESS

Students' home languages have been explored and discussed in details as important resources for teaching and learning especially as a means to improve bi/multilingual students' participation and performance in mathematics (Setati, 1998; Setati & Adler, 2000). In fact, research on the language practice of code switching, between the Language of Learning and Teaching (LoLT) and students' home language have been widely documented.

Code switching and Language proficiency

Code switching may be verbal (Baker, 1993) or non-verbal (Moschkovich, 2005) and can involve a word, a phrase, a segment of a sentence, a sentence or several sentences. The non-verbal strategy during solitary and/or mental arithmetic computation advanced by Moschkovich involves switching between languages when thinking through computations. This paper focuses on the non-verbal strategy when students individually engage in thinking and working on mathematics task.

Code switching is necessary when learners have limited proficiency in the LoLT; this is because limited proficiency in LoLT may prevent them from expressing their mathematics ideas in the LoLT and more particularly in mathematical language. Despite the fact that some students perform well in the LoLT, they also face interpretation challenges in mathematics, (Clarkson, 2006; Njurai, 2015). In such instances, it is observed that some students draw on their home languages to solve such problems. Code switching by students with limited proficiency in LoLT and those proficient in it presents an opportunity to find out how students explore mathematical meaning in tasks presented in LoLT. Code switching has however not been straight forward and dilemmas have been experienced.

The Dilemma of Code Switching

The language of teaching dilemmas developed by Adler (1998) provides an explanatory and analytical tool in multilingual mathematics education. Adler addresses three key dilemmas among them the dilemma of code-switching. The dilemma describes and explains mathematics teachers' knowledge of their practice in multilingual mathematics classrooms of South Africa. In this dilemma of code switching, Adler explores tensions between developing spoken mathematical English, where English is the LoLT, visa vie ensuring mathematical meaning. Embedded in this dilemma is the need to access mathematical concepts and at the same time access to English language, the language of power, furthermore, access to language of mathematics and mathematics discourse (Adler, 1998). The teachers were faced with a dilemma; to switch or not to switch. The dilemma was at once personal, practical and contextual.

Based on the language of teaching dilemma, researchers in mathematics education have explored and discussed the dilemma of code switching in multilingual mathematics classrooms from the teachers' perspective in diverse language contexts which include Malaysia, Malawi as well as South Africa (Barwell, Chapsam, Nkambulem, & Phakeng, 2016; Chitera, 2010; Lim & Presmeg, 2010; Setati & Adler, 2000). A question that begs response then is whether students who practice code switching experience the dilemma.

I find this notion of dilemma of code switching illuminating to describe and explain dilemmas experienced by trilingual undergraduate students of mathematics in Kenya when engaging with mathematics at an individual level.

THE TRILINGUAL CONTEXT

The Language of Learning and Teaching (LoLT) at university in Kenya is informed by the Language in Education Policy (LiEP). The policy states that during the first three years of their schooling, students in public schools are taught through the medium of the home language that is predominant in the school environment. The learners are introduced to learning their home languages as well as English and Kiswahili as subjects. The learners can then be described as trilingual learners. A trilingual person is one proficient in three languages and whose proficiency in the languages is not necessarily equal (Hoffmann, 2001). The speaker uses the three languages either separately or by switching between any two in ways that are determined by his/her communication needs. It is noteworthy that neither Kiswahili nor English are first or home languages for the majority of students. English is the official language while Kiswahili is the national language and co-official language (Republic of Kenya (RoK), 2010). While the three language formula is not implemented uniformly throughout the country in these early years, it recognises the value and importance of home languages while it progressively inducts learning in English only from the fourth year. At university, the LoLT is English for all non-language subjects. Even with the constitutional provision that both English and Kiswahili be used as official languages, the position of English as the LoLT is still dominant. It is the accepted language of teaching though it may not necessarily be the language of learning or thinking.

RESEARCH METHOD

This paper draws from a wider study which explored language practices of trilingual undergraduate students of mathematics in Kenya and which adopted a qualitative inquiry process, specifically a case study approach (Njurai, 2015). It was conducted in one public university in Kenya with a focus on first-year undergraduate of engineering students taking mathematics in their programmes. Data were collected using three instruments students' questionnaire, and clinical and reflective interviews.

The transcripts of reflective interviews, in which semi structured questions were used, are key in this paper in providing details of the code switching practices and the associated dilemmas. It is in them that the students talked about mathematics (Adler, 1998) explaining the different languages and language practices that were not apparent during the clinical interviews. The transcripts also provided data on how and why the participants used each language while processing the task, in speech in writing or other non-verbal means. The focus was on students who indicated that they code switched to translate part or whole task and expressed that in so doing they faced a dilemma.

The participants

The participating students were high performers in both mathematics and English. These were S10, S14 and S12, all taking a Bachelor's degree in Civil Engineering. Their home languages were Kikamba, Dholuo and Luluhya respectively and were aged between 19 and 21 years. A common thread was that the students needed to understand the mathematics task and saw the need to use their home languages to facilitate the understanding and interpretation. In their utterances, two students S10 and S14 indicated that they experienced the dilemma of to switch or not while the third student S12 offered a different view on the same.

Data Analysis and Findings

In working on the particular task, S10 used English throughout his written and spoken explanation of the task expectation, interpretation and solution process with the researcher. In his reflections, he revealed that he used Kikamba to interpret the whole task, because it was the more familiar language.

S10: Yeah, yeah, yeah. First, after seeing the question, in all my studies, I try to interpret in Kikamba, which I'm more conversant with. I read in English then I interpret it in Kikamba, which I can understand more than English.

R: Are there particular parts or it is the whole question that …. {S10 interjected}

S10: The whole question.

R: How do you put it in Kikamba?

S10: I do it in Kikamba then I transfer to the paper in English.

R: Is it {translation} something that you can write?

S10: No, no, no.

Yeah, I'm more conversant with Kikamba more than any other language.

This extract reveals that S10 not only translated the present task but that he did this with all other tasks. His reason was that he was more familiar with Kikamba than with English. He interpreted the task to Kikamba in his mind but neither verbalised nor wrote it down in this language. It was interesting that when requested to write the interpretation in his home language, S10 gave an emphatic "no", despite arguing that he was more fluent in this language than in any other. While this explanation may seem to contradict his use of Kikamba, in a way it demonstrates that conversational proficiency in a language is not commensurate with written proficiency. His home language was significant for understanding and interpreting the task.

His dilemma of code switching between English and Kikamba is explicit in the following extract.

S10: After seeing the question, first the question was very tricky, so I had to read it, reread it so that I can understand it more. Then in my translating to Kikamba and then to English, I think it wasted a lot of time.

R: Wasted time?

S10: Yeah…

R: You have said it helps you to understand it better?

S10: It helps but it wastes a lot of time.

R: What would be the option?

S10: If it is possible, I can try to practice to interpret the question in English which I use to write in paper.

He finds the process of translating back and forth as a process that wastes his time. To avoid time wastage, S12 observes that he needs to practice interpreting in English. This would call for him to be more familiar with English than he currently is. Therefore, his dilemma is to switch to interpret and understand the task hence taking more time or remain in the LoLT probably not get the required meaning while working in a shorter time of period.

In the task that was at hand, S14 used English only in his communication with the researcher. However, his reflections indicated that he translated part of the task into Dholuo[6]. In this extract, S14 had reworked the task after an incorrect first attempt.

S14: I involved it {Dholuo} at the stages where I was not able to interpret in terms of English.

S14: In part (b) I had to involve, I was a bit confused in terms of these people {450} and the number of seats here. I had to involve Dholuo and Kiswahili so that I interpret that each chair was supposed to

[6] While I have reported elsewhere that S14 switched between any two of his three languages, the current discussion engages in his switch to Dholuo where he faced dilemma of code switch.

accommodate an individual. So depending on the equation that I got in part (b), I had to equate to the number of people so that I could solve it.

R: How did it go like? If you can write.

S14: "Ka ji 450 obedo to gi wuoyo kombe, kombe ma odong' onego bed ni ting'o ji 150". [If 450 are seated and they rearrange the chairs, the remaining chairs should accommodate 150 people] {He then read out the translation}. I set out the equation for the remaining chairs... {inaudible}.

S14 had difficulties in the interpretation of the task in the second part and this caused some confusion. He needed to link the solution arrived at earlier to the requirements of the second part. In order to do so, S14 translated part of it into Dholuo and arrived at the solution. From S14's account, it is clear that Dholuo was used as a linguistic resource when he faced interpretation challenges in English.

S14 considered knowledge and use of a language in mathematics as important and that if one code switches, they must be careful to get the right interpretation.

S14: I think it [language] is a very important factor when it comes to mathematics. We have to comprehend the question and if you do the wrong interpretation especially when somebody switches from English and then brings another language let's say mother tongue or Kiswahili, it may bring a different interpretation apart from what was expected. So language is very important especially English is very important for mostly performing in mathematics.

R: Yes but it contradicts what you have already done; interpreting and translating to home language and you were able to succeed in that question…

S14: So what I can say when you now switch to other languages apart from English then it consumes a lot of time. If you understand English better, you can understand the question and then do it within the expected time.

S14 brings out two issues here: the possibility of misinterpretation and use of more time in translating. As observed by Setati (1998), mathematics registers may not be developed in all African languages as it is developed in the LoLT (commonly English or French in Africa). Therefore, the translation may not yield the expected interpretation. S14 notes that English is important for his working and performance in mathematics. This could simply not be the ordinary English but also mathematical English.

S14 also notes that a lot of time is used when code switching. Like S10, he suggests that if one is able to interpret the task in English, then one would be able to work on the task in the expected time period.

The need for more time by S10 and S14 resonates with the findings of Lim & Presmeg (2010) that, when code switching happens, then more time than is stipulated is required to translate both ways from LoLT to home language and back to LoLT.

A contrasting finding emerged with the analysis of data of S12. He explained that code switching is not a time wasting process but one that would save time. While his home language is Luluhya, he translated parts of the task into Kiswahili, the national language of Kenya and a language he commonly uses. He translated what he referred to as the general parts of the task into Kiswahili and the specifics he worked in English. The general parts appear to be the parts involving ordinary language and knowledge while the specifics involve mathematical language. In the latter, he was involved in making assumptions for the unknowns. In fact, in the following extract like S14 he says that English is key in understanding mathematics, but code switching to a more familiar language has its advantage of saving on time;

S12: …For you to be able understand the concept in mathematics, you have to be good in some other languages, but for this case, English is the key, then there are other languages which can help you to understand mathematics. …You know in mathematics we are tested on time and so many other things. So if you will not understand the concept may be in English or it will take you so long to understand, then you are killing your time. The best thing you can do is understand it maybe in another language so that you can minimize on that time waste for you to be able to handle the question.

S12 suggests that code switching to a language that one is familiar with is a prerequisite to understanding mathematical tasks. He suggests that if one does not understand a task in the LoLT, they can switch to their home languages; it is both easier that way and also saves on time.

DISCUSSION

The use of home languages does not seem to interfere or hinder expected meaning, in fact the languages are resources that provide opportunities for engaging with mathematical discourse. For all the students S10, S14 and S12, code switching facilitated understanding of the task; either partially or in whole. Their home languages were resources necessary in understanding and interpreting the task. S10 translated the whole task, S14 parts of the task while S12 translated the general parts but not those involving mathematical language. However, S10 and S14 sighted dilemmas at individual levels to switch or not to switch and waste time, and to lead into a possibility of misinterpretation. In contrast, S12 opines that by code switching one would save time by efficiently understanding the task in the language they are more familiar with.

It is evident from the findings above that while the students were proficient in LoLT, they needed to explore meaning in their other languages because of confusion, habitual practices and where the task involved general information (not mathematics specific). They needed their home languages to explore and access mathematical meaning of the task. This is in line with the finding by Clarkson (2006) that students who are proficient in LoLT also code switch to their home languages to explore mathematical meaning.

From the discussion, the notion of dilemma is apparent to learners who practice code switching as it is for teachers. The dilemmas are personal and interpersonal, and practical. The contribution of this study in the field of mathematics education and language diversity is that there are dilemmas that are language related (misinterpretation) or non-language (time wastage). Furthermore, in a classroom situation, misinterpretation may be corrected by the teacher, when students engage with tasks at individual levels, their interpretation in other languages is final and the misinterpretation are not corrected. The findings add to the on-going discussion on dilemmas faced in multilingual classrooms. Further research is recommended on dilemmas that students may be faced with and ways of dealing with them.

CONCLUSIONS

The dilemmas of code switching and exploring mathematical meaning in home languages takes significance in the context of curriculum reform in Kenya that is currently being piloted. Students' varying use of code switching and associated dilemmas suggests that language in-education policy needs to engage more seriously and explicitly with what bi/tri/multilingual practices like code-switching can and do mean in the day to day realities of trilingual students' engagement with mathematics.

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Using peer-mediated instruction to achieve equitable access to mathematics education in limited resource schools in Malawi

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To achieve equitable access to quality mathematics education, some learners require more support than others in the classroom. This is very challenging to a typical Malawian mathematics teacher who is often requested to implement equitable access to instruction within settings that have very limited resources. This paper discusses findings from two case studies that explored ways of achieving equitable access to education at the schools. The two cases were one primary school and one secondary school in Malawi. The case studies adopted a qualitative design involving learners, teachers, and specialist instructors for learners with disabilities at the two schools. It was found that both schools had mechanisms for supporting learners with visual impairment, hearing-impairment, and learning disorders. Among all the disabilities, hearing-impairment posed unique challenges because teaching mathematics to such learners largely depended on the competence of the specialist teachers. To mitigate this challenge, authorities at the secondary school equipped some capable mainstream learners in each class with minimum skills required to communicate with their hearing-impaired classmates during lessons, which is generic form of peer-mediated instruction (PMI). The findings suggest that PMI could be used to mitigate some of the challenges resulting from adoption of inclusive education.

BACKGROUND

The global shift towards inclusive education has made the teaching of mathematics in public schools to become more and more complex. In extreme cases, the teacher might experience contradicting instructional demands in the same classroom; such as, learners with visual impairment demanding more vocal than visual modes of instruction whereas those with hearing impairment prefer the opposite. Unfortunately, the mediation strategies explored during the standard training of a mathematics teacher predominantly involve verbal talk and chalkboard inscriptions; thereby assuming that all learners can hear what is being said by the teacher and can see what is being written on the chalkboard. In the absence of adequate training and technical support to a typical teacher, coping mechanisms have to be devised by the concerned teachers in public schools to comply with government legislation and policies demanding enrolment of learners with special education needs.

In order to understand the experiences of mathematics teachers after the adoption of inclusive education, this research work focused on a prevalent, yet hidden disability, hearing impairment (HI). During the 2014/2015 academic year, about 26 percent of learners with special needs in Malawian primary schools had HI (MoE, 2016). This paper specifically focuses on leaners with total hearing loss which occurred before language acquisition, who prefer the use of sign language, and are also referred to as "culturally Deaf"— with an uppercase ― d (Gao, 2007). Research has shown that, on average, individuals who lost their hearing before language acquisition lag behind their peers in language comprehension by as many as 8 years by the time they are completing their secondary school education (Garberoglio, Dickson, Cawthon, & Bond, 2015). This is a very wide gap. This poses a big challenge to the mathematics teacher, considering that mathematics in itself is a language which is often taught through a second or third language. Even if the mathematics teacher would wish to teach learners with HI in sign language, Malawi has not yet fully standardised its national sign language, hence it is mainly in community based "dialects".

The research attempted to answer the following questions:

- What strategies are being used in Malawian public schools to mediate delivery of instruction to people with hearing impairment?
- To what extent can peer mediated instruction be used to ensure equitable access to quality mathematical instruction?

THEORETICAL FRAMEWORK

The research was guided by sociocultural theory originating from the work of Lev Vygotsky, which states that learning is a social phenomenon and takes place in specific cultural, historical and institutional contexts (Hyland & Hyland, 2006). The social model of disability was used to understand the experiences of people with HI, which asserts that the society originates disability by regarding some things and actions as normal and others as abnormal, hence the society can eliminate disability by developing barrier free environments (WHO, 2001). This paper discusses a public secondary school which created a barrier free school environment for learners with HI through the adoption of peer-mediated instruction.

Peer-mediated instruction (PMI)

Peer-mediated instruction (PMI) is a collaborative learner centred teaching strategy in which some learners take the role of an instructor to their classmates. The PMI strategy has a large research evidence base spanning decades. It has mainly been used for enhancing social interaction skills of passive learners, especially during early years of schooling, but has also proved to have positive outcomes for older learners. A meta-analysis of several studies assessing the use of PMI on the teaching and learning of mathematics found that the strategy was effective for improving performance in mathematics for individuals with learning problems (Kunsch, Jitendra, & Sood, 2007). Some studies on peer-mediated strategies indicate that the intervention promotes academic achievement for both the peer tutor and tutee (Hott, Walker, & Sahni, 2012).

One alternative to PMI is technology-mediated instruction (TMI), requiring a huge upfront investment in resources and skills, which are not currently available in an average Malawian public school.

METHOD

The research adopted a qualitative case study design involving two cases, one secondary school and one primary school located in the same area. The secondary school was purposively selected as a paradigmatic case, for exemplifying the use of PMI, while the primary school served as a typical case (Palyst, 2008). The research examined the inclusive practices at each of the two schools but this paper will mostly refer to the paradigmatic secondary school case. Data was initially collected through classroom observations and later authenticated through follow-up interviews with mathematics teachers, school administrators, learners with HI, hearing peer mediators, and specialists for learners with special needs. Some insights were noted from questionnaires which were administered to the secondary school learners during a parallel baseline study involving the same students and researcher. During data collection, the secondary school had a total of 18 learners with HI while the primary school had 9 learners.

The data collected from the different approaches underwent thematic data analysis. Patterns across the data sets were analysed for common themes which appeared to be sufficient to answer the research questions.

FINDINGS

When carrying out the research, it was found that each of the two cases had one resource room and at least one active specialist teacher for learners with special needs, but used different support structures for learners with HI.

Strategies for inclusion of learners with HI in mathematical instruction

For the secondary school case, the standard arrangement was that after regular school hours, learners with HI had to seek remedial instruction from the specialist teachers in a resource room for learners with special needs. However, only one of the three specialist teachers was active, but she was neither a specialist in HI nor was she a mathematics teacher. As such, she relied on senior learners who developed HI after language acquisition to act as peer mediators for relaying remedial mathematical instruction to those who relied on sign language, or just referred them back to their mathematics teacher. The exception was the primary school specialist who was able to handle remedial teaching of mathematics and other subjects, but the arrangement required withdrawing all learners with special needs from their respective mainstream classrooms to the resource room during regular school hours, thereby making them miss other lessons.

The secondary school noted that supporting learners with HI outside the regular classroom could not solve problems experienced during normal teaching. Oftentimes, there was frustration both on the part of the teacher and learners with HI. During an interview, one mathematics teacher indicated that he felt helpless when handling an inclusive class, saying: *"We expect the Government to send us learners with impairments but the severity of impairments is not as expected"*. His expectation was that the learners with HI selected to the school should have some capabilities like lip-reading and good comprehension of written text, but the reality was that most of them mainly relied on sign language for communication. On the other hand, some learners with HI complained that some class teachers seemed not concerned with their presence in the classroom by talking while facing the chalkboard and writing very limited text. Since HI is invisible, even the conscious teachers would sometimes forget to mind such learners.

Adoption of PMI

To address the challenges with sole use of specialist teachers for learners with HI, the secondary school designated inclusive classes at each level and decided to equip some capable mainstream learners with minimum sign language skills required to become peer tutors of classmates with HI (as tutees) during lessons. The peer tutors were progressively taught sign language by the tutees. At the outset, the entire school was also given basic orientation on sign language. The positive results of this arrangement were highlighted by the peer tutors, tutees, and school administrators. In addition to bridging the knowledge divide among the learners, school authorities also noticed that the PMI strategy resulted in social inclusion of learners with HI, who formerly socially isolated themselves in small groups.

However, since the peer tutors had never undergone formal training in sign language interpretation, those in the lower secondary school classes had less experience and had very limited skills. The peer tutors also indicated that they were overtaxed during instruction, because they are expected to comprehend a new concept and be expected to simultaneously relay it to a tutee. In some cases, the peer tutees seemed uncooperative. One peer tutor stopped assisting a tutee following disagreements between the two, making this voluntary PMI approach unreliable. In one class it was observed that the mathematics teacher had to remind a learner with HI to sit in front, but the learner was reluctant to move from the current seat, stigmatising the front row as "remedial".

IMPLICATIONS OF FINDINGS ON MATHEMATICS EDUCATION

The findings of this research work suggest that PMI could be employed to achieve inclusion of learners who rely on sign language in the mathematics classroom. The major strength of the approach is that the peer tutor and the tutee can build their unique vocabulary of signs for newly encountered mathematical terminologies in the absence of a standardized national sign language. Teacher training institutions could also consider equipping pre-service mathematics teachers with skills on various possibilities of using PMI in the classroom; but stressing the need for proper training, monitoring mechanisms, and continuous support to peer tutors.

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Sub-theme 3

Mathematical thinking for nurturing quality education

The co-emergence of visualisation and mathematical reasoning in word problem solving

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This paper reports on the second phase of data collection and analysis of the bigger PhD study that sought to examine the co-emergence of visualisation and reasoning processes in the context of geometry word problems. Participants were eight mixed-gender and mixed-ability Grade 11 learners from a private school in southern Namibia. In small groups, the participants responded to selected five task-based interview questions by making use of visual imagery and reasoning processes to reach a collective solution. Key enactivist concepts of co-emergence and structural coupling provided the language to discuss the relationship between visualisation and reasoning during data analysis. Findings indicated that the visualisation processes enacted by the participants while solving geometry word problems in collaborative groups were inseparable from the reasoning processes. They were closely interlinked throughout the task-based interviews – that is, they co-emerged.

Keywords: Co-emergence, reasoning processes, visual imagery, visualisation processes

INTRODUCTION

As a mathematics teacher for many years, it has always been a concern that despite the accuracy of my learners' responses to mathematical problems, many of them struggle to provide reasons for their solutions. The most common responses I get from these learners include: 'I know what the answer is, but I don't know how I got it', 'I used a calculator, can I show you?', 'I don't know how to say it, but I did it', 'I don't know how I got the answer, I just know that it is right'. Such responses have been worrisome over the years and have hence inspired me to delve deeper into the matter. The main purpose of the bigger PhD study was to analyse how visualisation processes related to reasoning processes during collaborative groups. This paper sets out to report on the findings of that relationship. The question that guided the second phase of the study was:

How do visualisation and reasoning processes co-emerge when learners solve geometry word problems in small collaborative groups?

LITERATURE BACKGROUND

Co-emergence

Co-emergence as interpreted by Li, Clark and Winchester (2010, p. 407) refers to a situation whereby a change of both, a living system and its surrounding environment depends on the interaction between the system and its environment. When the system and the environment interact, they become structurally coupled. This means that the mutual interaction of the organism and the environment causes changes and transformations in both (Khan, Francis, & Davis, 2015). The study is underpinned by the enactivist perspective of human cognition. Begg (2013, p. 82) claims that in the enactivist perspective, humans and the world are inseparable: they co-emerge. As a result, cognition (learning) cannot be separated from being (living), and knowledge is the domain of possibilities that emerges as we respond to and cause changes within our world. Thus, one cannot separate knowing from doing and from the body. Brown (2015) stresses that "we are co-emergent and where there is a coordination of actions, like in a classroom, or a collaborative group in a research project, a culture of practices emerges that is good-enough (effective action) to get done what needs to be done" (p. 188). Li et al. (2010, p. 407) caution that while co-emergence suggests that the system and the environment interact, it does not guarantee greater or lesser adaptation on the part of either to each other. Students bring forth a world; they emerge with it, but it is their structures that bring them forth (Proulx, 2008a, p. 22). This inseparability of body, mind and environment is known as embodied cognition (Alibali & Nathan, 2012; Antel, 2009; Wilson, 2009).

Mathematical reasoning

Mathematical reasoning in our study refers to the process that involves forming and communicating a path between one idea/concept and the next (Brodie, 2010, p. v). In our study, we made use of four reasoning processes to tease out the patterns of reasoning in the participants' responses to selected geometry word problems. The four reasoning processes (RPs) are explanation, justification, argumentation and generalisation. **Explanation (RPE)** refers to the classification aspects of one's mathematical thinking that he/she thinks might not be readily apparent to others. **Justification (RPJ)** is defined by Staples, Bartlo and Thanheiser (2012) as "an argument that demonstrates (or refutes) the truth of a claim that uses accepted statements and mathematical forms of reasoning" (p. 448). **Argumentation (RPA)** is defined by Lithner (2000) as the "substantiation, the part of reasoning that aims at convincing oneself, or someone else, that the reasoning is appropriate" (p. 166). **Generalisation (RPG)** refers to the process of identifying operators and the sequence of operations that a common among specific cases and to extend them to the general case (Swafford & Langrall, 2000).

Visualisation

Visualisation as defined by Arcavi (2003) refers to:

> the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding. (p. 217)

Visual imagery (VI) is used to unpack visualisation processes in our study as well as for data analysis purposes. Five categories of VI are defined as follows:

Concrete pictorial imagery (CPI) – this refers to the concrete image(s) of an actual situation formulated in a person's mind – i.e., a picture in the mind, drawn on paper or described verbally.
Pattern imagery (PI) – this refers to the type of imagery in which concrete details are disregarded and pure relationships are depicted in a visual-spatial scheme. The essential feature of pattern imagery is that it is pattern-like and stripped of concrete detail Presmeg (1986).
Memory imagery (MI) – this refers to the ability to visualise an image that one has seen somewhere before. This too includes a history of recurrent occurrences.
Kinaesthetic imagery (KI) – this is the kind of imagery that involves muscular activity. A kinaesthetic visualiser wants to feel and touch.
Dynamic Imagery (DI) – this imagery involves processes of transforming shapes i.e. redrawing given or initially own drawn figures with aim of solving the problem.

RESEARCH METHODOLOGY

The methodology discussed in this paper is that of the second phase of data collection and analysis of the bigger PhD study. The purpose of the second phase of this qualitative case study was to analyse the relationship between visualisation processes and reasoning process when the eight participants, divided into three small collaborative groups solved geometry word problems (GWP) during the focus groups task-based interviews.

The task-based interviews were transcribed, and the data was analysed using the two analytical tools; first for the visual imagery, and then for reasoning processes. I was both interested in individual learners' reasoning processes and as a group. Hence, data was initially analysed for individual reasoning and visualisation processes, and then summarised in a matrix coding by making use of the NVivo Software for each focus group. The matrix coding was primarily done to analyse the relationship between each reasoning process and the five categories of visual imagery (5VIs) that was observed during task-based interviews (see Figure 2 below). This was followed by a fine-grained analysis that took the form of eight vignettes selected from the focus groups' transcripts for each of the 4RPs. One of these vignettes is discussed later in this paper.

RESULTS

Having already analysed the data using the 5VIs, I interweaved the 4RPs with the 5VIs and concluded the analysis with a discussion of the co-emergence of visualisation and reasoning processes. Figure 2 illustrates an overview of the results of the coding matrix of this co-emergence.

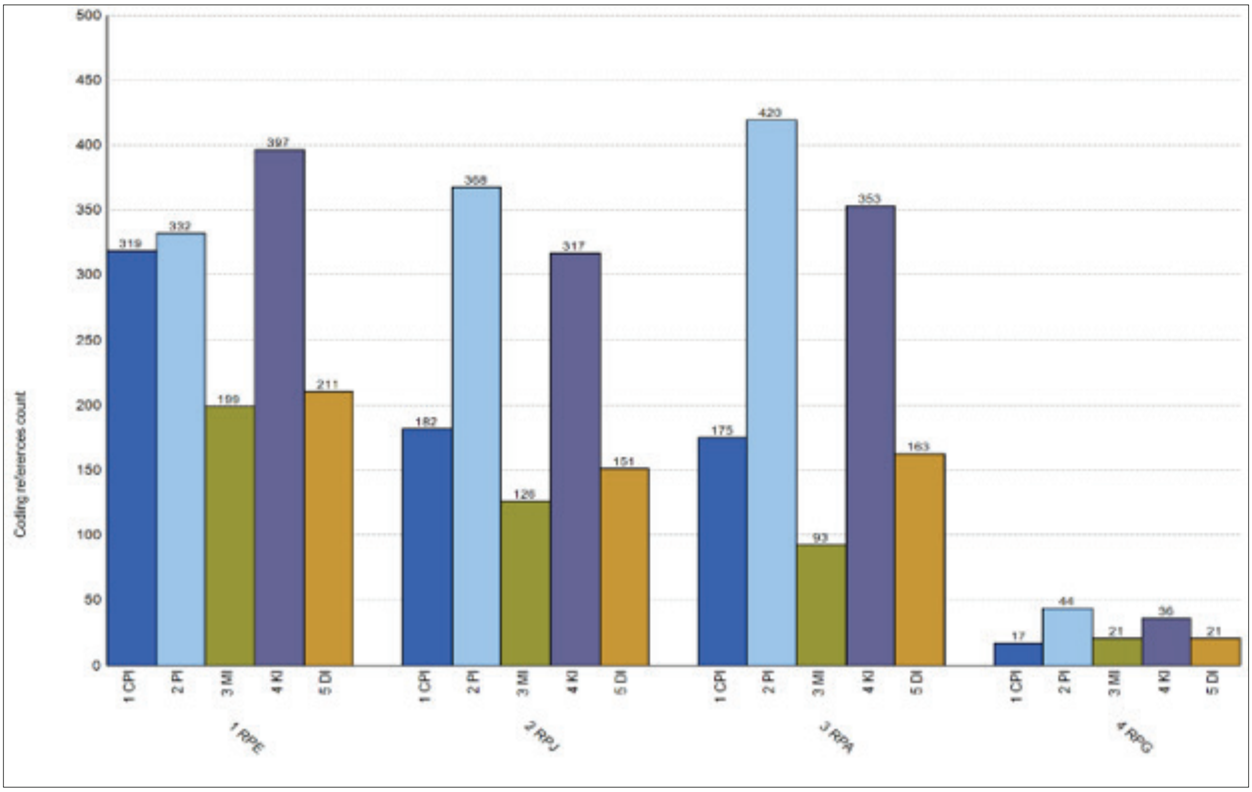


Figure 2: Overview of the data analysis matrix for the relations of visualisation and reasoning processes

As illustrated above, the closest relationship between the 5VIs and the 4RPs was recorded between the pattern imagery and the reasoning process of argumentation. This means that most of the research participants formulated patterns with the purpose of communicating information, engaged in patterns of data and argument, and used visualisation to venture generalisations, providing proofs, explanations and justifications, while convincing/persuading others of the truth of their claims and accepting/refuting the truth of others' claims at the same time. There was also a strong connection between kinaesthetic imagery and the reasoning process of explanation, with a matrix coding of 397 references. Pattern imagery also recorded a close relationship with the reasoning process of justification (RPJ), where participants provided proofs to validate their claims, provided/sought rationales for actions taken, as well as promoted understanding among those engaged in a justification. These recorded a matrix coding of 368 as illustrated in Figure 2 above.

In this paper, I discuss the co-emergence of the RPJ and the 5VIs when one focus group that consisted of three girls, Millie, Meagan and Rauna solved the first task of the GWP worksheet. The first task that was posed in the worksheet is:

Task 1

Marina's backyard is a square with a side length of twenty meters. In her backyard is a circular garden that extends to each side of her yard. In the centre of the garden is a square patch of spinach so big that each corner of the square touches a side of the garden. Marina really likes spinach! How much area of Marina's garden is being used to grow spinach?

The girls started by making a rough sketch to represent the information in the task. There however, erupted a disagreement between Millie and the other two girls which led to further visualisations and more reasoning as well as the co-emergence of the two as the girls tried to find a consensus, and to eventually solve the task. Millie drew a neat sketch to represent Task 1 (Figure 3) as she explicated her reasoning.

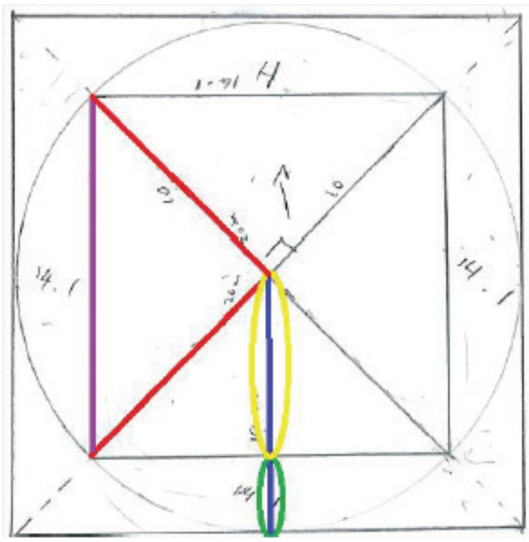


Figure 3: A not-to-scale sketch to represent Task 1 drawn by Millie

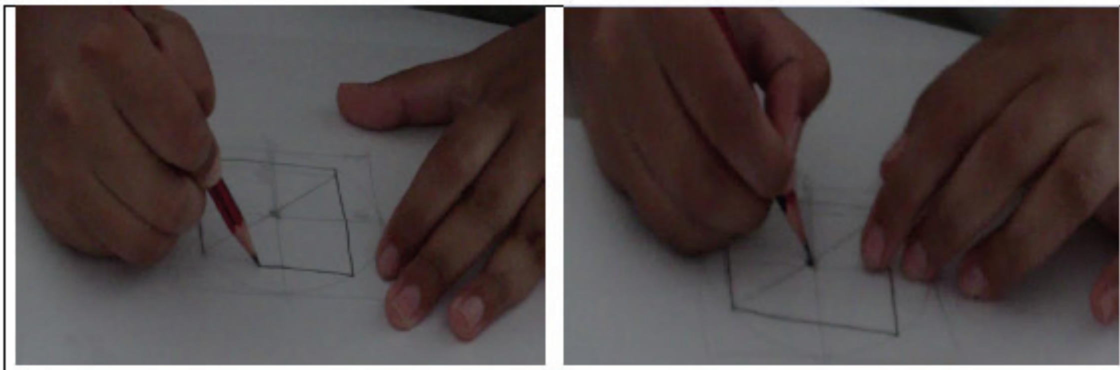
The excerpt below illustrates Millie's support for her argumentations:

> *So, you guys are saying basically, that from here till … each point of this square* [points the vertices of the square with a pencil] *is touching the circle; the circumference* [imitates drawing a circle by gestural movement with a pencil in the air]. *And, what I'm saying is from here till here,* [refers to the red lines in Figure 3] *it can be…it's also ten centimetres if the circle radius is ten centimetres* [walks a path with a pencil on the radius of the circle – the blue line].

When Rauna and Meagan disagreed with her, Millie continued with her explanation and provided proofs for her claims as follows:

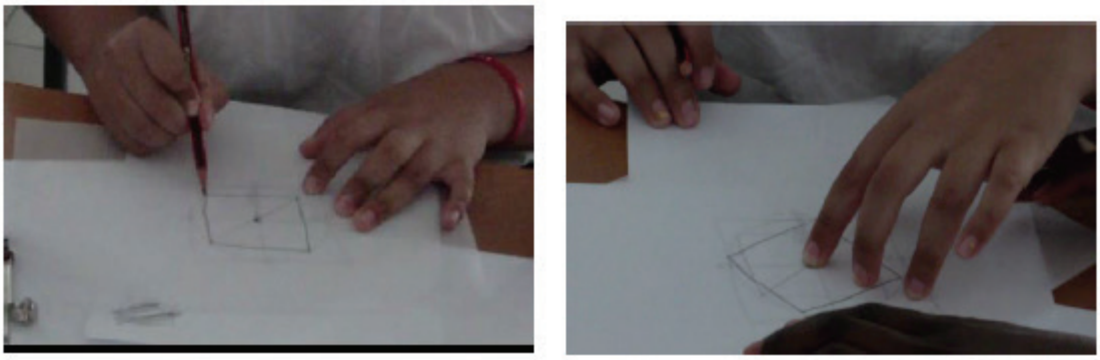
> *Because if you turned it…if you turned it…* [holds the centre with a finger and turns the tracing paper such that the length between the centre and the vertex of the square equals the radius of the circle*] if the…because it doesn't look like this, it's not accurate, but if it was a perfect square, it would still be the same* [gestural movements with her hand as she justifies her point].

In addition to oral justifications, Millie incorporated gestures and drawings to vividly present her arguments (Figure 4). This is in line with Antel's (2009) observation that the structure of the human body acting in complex physical, social and cultural environments determines perceptual and cognitive structures, processes and operations. Further, justification as a learning practice promotes understanding among those engaged in the justification – both the individual offering a justification (in this case, Millie) and the audience of that justification (Meagan and Rauna) (Staples et al., 2012).



(a) "we are drawing a square on here"

(b) "let's first show where the radius is"



(c) "so what I'm saying is from the centre of the square to any of its four points"

(d) "because you can rotate it…" [placed her finger at centre of rotation]

Figure 4: Millie's tracing paper

Millie eventually used a tracing paper (Figure 4) to show the other girls why she claimed that the half of the diagonal of the inscribed square (red in Figure 3) equalled the radius of the circle in which it is transcribed (blue in Figure 2), as they were both radii of the circle. Millie's use of a combination of VIs to demonstrate her reasoning helped to convince Meagan and Rauna to accept her arguments and justifications as well as to improve their understanding.

CONCLUSION

When Meagan and Rauna initially resisted Millie's arguments (including that of Figure 3), she opted for a more visual method to enhance her teammates' understanding when she used the tracing paper. She used visualisation processes to help her improve Meagan and Rauna's understanding of the relationship between the radius of the circle and the half of the diagonal of the inscribed square. The more Millie justified her arguments, the better she perfected her mathematical reasoning. From an enactivist perspective, Millie's interaction with her living and active body created her structural couplings with the other two girls. This interaction further created co-emergences of Millie's and her teammates' VIs and RPs, which in return produces the "structural coupling" that enabled them to continue interacting (Rossi, Prenna, Giannandrea, & Magnoler, 2013, p. 38). The cycle is repeated as the understanding improves in the group. The co-emergence of VIs and RPs then becomes more apparent.

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First year students perception of the Use of Symbols in mathematics learning

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This study investigated how the use of mathematical symbols influences understanding of mathematics concepts by students taking mathematics at Nkhoma University. The symbols used in the learning of set theory were conveniently sampled for this study. A sample consisted of all students studying mathematics as one of the modules in their programmes of study. A questionnaire consisting of items on meaning and understanding of symbols, use of symbols and perception of the influence of symbols on students' performance in mathematics was developed and administered to 292 students taking mathematics in their programme of study. Data was analysed using SPSS for items that required ratings and diagnostic questions were marked either correct use of symbols or wrong use together with correct or wrong answers. It was found that most students fail to understand or interpret the meaning of mathematics symbols due to the way they are taught to read, pronounce and use them. The majority of students believed that mathematics is all about symbols. Lecturers and textbooks do little to help students acquire the mathematics symbols. Consequently, students believe that mathematics is hard partly because of the use of symbols which in most cases are not properly explained.

Keywords: Symbols, symbolism, notation, misuse, abuse, influence, mathematics concepts

INTRODUCTION

This study is drawn from the field of mathematics symbology. In this modern technological world Mathematics plays a greater role than ordinary (O'Halloran, 2005) in trying to find solutions to everyday problems (Esty, 2011). Coding and decoding information, shortening sentences and representing and analyzing data are all processes where mathematical symbols are used. Mathematics is also itself a language with an internationally recognized syntax and vocabulary (Esty, 2011). Symbols are the components of the mathematics language that make it possible for a person to communicate, manipulate, and reflect upon abstract mathematical concepts (Goldin & Sheingold, 2001). However, the symbolic language is often a cause of great confusion for students (Rubenstein & Thompson, 2001). The expert mathematician or mathematics lecturer is able to work with and to "see" the mathematics through its symbolic representations, whereas students often struggle in this endeavor; they may need to be told what to see and how to reason with mathematical symbols (Bakker, Doorman, & Drijvers, 2003; Kinzel, 1999; Stacey & MacGregor, 1999). The way in which Mathematics exploits the spatial features of its

symbolisms and develops manipulations of symbolic expression is a special property not shared with ordinary languages such as English or Chichewa (Kaphesi, 2002).
While teaching Mathematics, the researcher found out that most students failed to grasp mathematics skills and concepts. The typical approach for helping students who are failing is to provide them with more practice problems. However, if instructors are not aware of students' networks of understanding, more practice may only reinforce misunderstanding (DeMarois, 1996; Dubinsky, 1991). It is possible that some students are actually developing different, incorrect techniques for problem solving due to their personal interpretations of the symbols involved (Gray & Tall, 1994). Since many studies have been conducted on exploring the possible explanation for students' failure to grasp mathematical concept, very few studies have been done to explore if failure could have been that the symbols which they encountered were unfamiliar, confusing and sometimes contradictory. The researcher then became interested in finding out the truth about this conjecture, focusing on the topics on set theory.

Statement of the problem

All Mathematics deals with symbols and notation and that books and teachers always have to explain their use to students. In learning mathematics, students have to deal with symbols. Problems on the misuse (and also abuse) of mathematical symbols have always been there since people started counting. Introduction of symbols replace special words, abbreviations, and number symbols which are a common style. The pressure to the introduction of symbols comes from the expanding scientific demands on Mathematics.
However, the deliberate use of symbolisms are not incidental or accidental, and the understanding and interpretation of the symbols in mathematics are imperative. The introduction of symbols into mathematics is not accidental. The problem then, is that far too many symbols have so far been introduced. Since the problems of inventing, using and interpreting symbols have always been there alongside other developments in mathematics, confusion and mistrust among mathematicians have resulted in many symbols not being standardized. The terms and notation vary a great deal; many symbols are derived from abbreviations. But as far as one can judge, the introduction of letters for classes of numbers has been accepted as a minor move in the development of symbolism.
Today the problem is no longer on inventing symbols, but on using, reading and interpreting them because some symbols basic mathematical concepts and call for operations. However, the challenges are on the use of mathematical symbols and how symbols influence understanding and mastery of concepts. The problem is that the symbols themselves are taken as the objects of mathematics rather than the ideas and processes which they represent. Learners fail to interpret or understand the meaning of certain mathematical symbols due to the way they are taught to read those symbols. Learners studying alone at home usually do not know how to read many mathematical symbols because they seldom hear them being spoken. A lot of research on how the use of mathematical symbols influence understanding of concepts has been carried out at the primary school level, probably very little at secondary school level and virtually nothing at university levels. Yet, more omplex symbols are introduced to students as they progress to tertiary and higher education. The general consensus is that the introduction of mathematical symbols presents difficulties and challenges

beyond those presented by words alone at all levels of education (Kuster 2010, Lee 2004).

Purpose of the Study

The purpose of the study is to investigate the influence of mathematical symbols on mathematics learning. It is also intended to sensitize mathematics educators to problems or challenges that students often have with mathematical symbols and to suggest instructional strategies that can reduce such difficulties since using symbols fluently and correctly is a necessary condition for overall mathematics achievement (Rubenstein & Thompson, 2001).

Research questions

The research question addressed is: How do first year students perceive the use of symbols mathematics learning? Specifically, the study addresses the following questions:

(a) What is the students understanding of mathematical symbols?
(b) How do students learn mathematical symbols?
(c) How do students use mathematical symbols?
(d) How do symbols influence mathematics learning?

REVIEW OF LITERATURE

A symbol is something that represents, stands for, or suggests an idea, belief, action, or entity. The word “symbol” comes from the Greek word for “token,” or “token of identity,” which is a combination of two word-roots, sum (“together”) and the verb ballo (“to throw”). A more relaxed interpretation would be “to put together.” Its etymology comes from an ancient way of proving one’s identity or one’s relationship to another (Miller, 2004).

In the context of this study, the term symbol refers specifically to mathematical symbols. These include letters, numbers, equal signs, plus and minus signs, parentheses, square root signs, etc. (Arcavi, 1994) and as such symbolic representations involve manipulations of symbols. Arcavi (1994) and Stacey & MacGregor (1999) have identified the underlying understanding of mathematical symbols and their uses as symbol sense, which Arcavi (1994) explains as “a quick or accurate appreciation, understanding, or instinct regarding symbols” (p. 31) that is involved at all stages of mathematical problem solving. Working fluently with symbols in mathematics requires developing strong symbol sense. Arcavi (1994) does not attempt to formally define symbol sense, claiming that to do so is difficult because it interacts with other senses such as number sense or function sense, but instead provides an extensive list of examples of what it might mean to have symbol sense. Arcavi (1994) suggests that many students fail to see mathematics and it’s symbols as a tool for understanding, communicating, and making connections, and he sees development of symbol sense as a necessary component of sense-making in general in mathematics. It is a tool that allows students to read into the meaning of a problem and to check the reasonableness of symbolic expressions.

Perhaps there should be a distinction between symbols and notation. Notations come from shorthand, abbreviations of terms. If symbols are notations that provide us with subconscious thoughts, consider “+.” Alone, it is a notation, born simply from the shorthand for the Latin word et. It was meant to denote a mathematical operation as well as the word "and." Numerals and all nonliteral operational notation are different, but still considered symbols, for they represent things that they do not resemble. Read the statement 5 + 3 = 8. It is a complete sentence in mathematics, with nouns, a conjunction, and a verb. It took you about a second to read it and continue on. Unaware of your fact-checking processes, you believe it for many reasons, starting from what you were told as a young child and ending with a mountain of corroborating evidence from years of experience. You didn't have to consciously search through your mental bank of truthful facts to know that it is true.

Yet there is a difference between the writer's art and the mathematician's. Whereas the writer is at liberty to use symbols in ways that contradict experience in order to jolt emotions or to create states of mind with deep-rooted meanings from a personal life's journey, the mathematician cannot compose contradictions, aside from the standard argument that establishes a proof by contradiction. Mathematical symbols have a definite initial purpose: to tidily package complex information in order to facilitate understanding (Stacey & MacGregor, 1999). Writers have more freedom than mathematicians. Vergani (1998) points out that literary symbols may be under the chains of myth and culture, but they are used in many ways. Some symbols also tend to evoke subliminal, sharply focused perceptions and connections. They might also transfer metaphorical thoughts capable of conveying meaning through similarity, analogy, and resemblance, and hence are as capable of such transferences as words on a page.

The argument here is whether it is possible to do all of mathematics without symbols. Words in a natural language such as English or Latin can present tight meaning, but almost never precision the way symbolic algebra can (Schleppegrell, 2007). Imagine what mathematics would be like if it were still entirely rhetorical, without its abundance of cleverly designed symbols. Even as late as the early sixteenth century, mathematics writing in Europe was still essentially rhetorical, although for some countries certain frequently used words had been abbreviated for centuries. The abbreviations became abbreviated, and by the next century, those abbreviations became so compacted that all the once-apparent connections to their origins became lost forever (Meaney, 2005; O'Halloran, 2005).

According to Schleppegrell (2007), the symbolic form of a rhetorical statement in mathematics is more than just convenient shorthand. First, it is not specific to any particular language; almost all languages of the world use the same notation, though possibly in different descriptory forms. Second, and perhaps most importantly, language helps the mind to transcend the ambiguities and misinterpretations dragged along by written words in natural language. Mathematicians often communicate in sequentially symbolic messages, a code, unintelligible to the uninitiated who have no keys to unlock those briefcases full of meaning (Pimm, 1995). They lose the public in respect of marks, signs, and symbols that are harder to learn than any natural language humans have ever created (Schleppegrell, 2007). More often, in speaking, for the sake of comprehension, they relax their airtight arguments at the expense of mildly slackening absolute proof. They rely on what one may call a "generosity of verbal semantics," an understanding of each other through a shared essence of professional expertise and experience independent of culture (Vergani, 1998).

The History of Mathematical Symbols

Use of mathematical symbols is chiefly a history of mathematical symbols; however, it is also an exploration of how symbols affect mathematical thought, and of how they invoke a wide range of enduring subconscious inspirations (Miller, 2004). Galileo is reported as having described Mathematics as the language with which God wrote the Universe (Cajori, 1993). He was correct in calling mathematics a language, because like any dialect, mathematics has its own rules, formulas, and nuances. In particular, the symbols used in mathematics are quite unique to its field and are deeply rooted in history (Miller, 2004).

According to Piaget (2001), Mathematical abstraction began as a process of extracting the underlying essence of a mathematical concept, removing any dependence on real world objects with which it might originally have been connected, and generalizing it so that it has wider applications or matching among other abstract descriptions of equivalent phenomena. Abstraction of notation is an ongoing process and the historical development of many mathematical topics exhibits a progression from the concrete to the abstract. Various set notations would be developed for fundamental object sets. In the history of mathematical notation, ideographic symbol notation has come full circle with the rise of computer visualization systems (Miller, 2004).

The ability to understand and predict the quantities of the world is a source of great power. Currently, that power is restricted to the tiny subset of people comfortable with manipulating abstract symbols. By comparison, consider literacy. The ability to receive thoughts from a person who is not at the same place or time is a similarly great power. The dramatic social consequences of the rise of literacy are well known. Linguistic literacy has enjoyed much more popular success than mathematical literacy. Almost all "educated" people can read; most can write at some level of competence. But most educated people have no useful mathematical skill beyond arithmetic.

Writing and mathematics are both symbol-based systems. But I speculate that written language is less artificial because its symbols map directly to words or phonemes, for which humans are hard-wired. I would guess that reading ties into the same mental machinery as hearing speech or seeing sign language.

Rationale for Use of Symbols in Mathematics

Pimm (1995) recognises the importance of symbols and suggests that mathematical applications can be effective if the calculations are summarised and structured by means of good notations. The proper use of (good) notations can be understood and put into effect if one knows the features or principles employed in mathematics symbolism. Some of these are order, position, relative size, orientation and repetition. Nevertheless, Pimm (1987) argues that when one learns mathematics, it is doubtful whether learners can distinguish between the mathematics learned and its symbols. He further suggests that the Mathematics symbols are not symbols, properly speaking, because symbols stand for something which they themselves are not; hence the misuse (or abuse) of symbols in mathematics teaching. Probably no one, however, has done extensive research on the use of these symbols.

It is undeniable that symbols not only enhance understanding but also provide a universally perceivable manner in which to show a certain mathematics function or illustrate a sequence (Stacey & MacGregor, 1999). The fundamental need in mathematics is to represent the relationship between a symbol and the concept it refers. Certain concepts can be clearly illustrated only by the creation and use of symbols. Measuring the relationship between numbers and representing the relationship symbolically not only serve to simplify the process but also gains a better understanding of the concept than a wordy description of the same (Chirume, 2012). This is where the issue of language comes in.

In more simple terms, a plus sign, a minus sign, a multiplication sign are all symbols. We need them for a very simple reason: we have to express what we are doing in a clear manner. When we are adding it would be ridiculous to always write out one plus one equals two when we could express this symbolically with 1+1=2. Imagine trying to perform calculus if you have to write a lengthy equation out in several paragraphs (Aspinwall, 2007). Not only would such prose be voluminous. It would be confusing and prone to error. Mathematics is universal but languages are incredibly vast. Simply put, without proper symbols, mathematics becomes next to impossible. In fact, you could look at this way: the symbols of mathematics are reflective of a mathematical language.

Mathematics is comprised of primarily two things: concepts and symbols (Pimm, 1995). Symbols are found in simple Mathematics and they are essentially representative of a value. It is important to understand that the key to comprehending Mathematics is in the interpretation of the concept and not really in the nature or amount of symbols and the role they play. However, to understand concepts, one must essentially have a good grasp of the role and meaning of symbols and also be able to appreciate their usefulness in making Mathematics that much simpler to understand and duplicate. The symbols in Mathematics are undeniable and are a vital tool in making Mathematics a universal science. They are sometimes taken for granted because symbols are so common in Mathematics and that they make Mathematics so easy to perform. Without various symbols, you would be forced to go back to counting your fingers and toes. Many people have come to believe mathematics is the memorization of, and mastery at using various formulas and symbolic procedures to solve encapsulated and essentially artificial problems. Such people typically have that impression of Mathematics because they have never been shown anything else.

Although ordinary people can do every day Mathematics (Stacey & MacGregor, 1999), they cannot do symbolic everyday Mathematics. Only someone who has mastery of symbolic mathematics can recognize the problems encountered in the two contexts as being "the same." Symbols are meaningless without understanding. Symbols don't mean anything unless you understand the concept. The symbol for '5' means nothing unless you associate it with the concept of 'five-ness' (Stacey & MacGregor, 1999).

Difficulties with Mathematical symbols

Mathematical symbols can be confusing and can act as a real barrier to learning and understanding basic numeracy (Chirume, 2012). One fairly common difficulty experienced by people with Mathematics problems is the inability to easily connect the abstract or conceptual aspects of Mathematics with reality. Understanding what symbols represent in the physical world is important to how well and how easily a child will remember a concept.

A far less common problem and probably the most severe, is the inability to effectively visualize Mathematics concepts. Kinzel (1999) observed that students who have this problem may be unable to judge the relative size among three dissimilar objects. This disorder has obvious disadvantages, as it requires that a student rely almost entirely on rote memorization of verbal or written descriptions of Mathematics concepts that most people take for granted. Some mathematical problems also require students to combine higher-order cognition with perceptual skills, for instance, to determine what shape will result when a complex 3-D figure is rotated.

Some people might argue that all Mathematics deals with symbols and notation and that books and lecturers always explain their use to students, hence there is no problem at all. This might not be true. Problems on the misuse (and also abuse) of mathematical symbols have always been there since man started counting. According to Cajori (1993), Diophantus introduced symbolism before the 16th century to replace special words, abbreviations, and number symbols which were a common style in the Renaissance. The pressure to introduce symbols in the 16th century came from the expanding scientific demands on Mathematics. By the end of the 17th century the deliberate use of symbolism, not incidental or accidental, and the understanding and interpretation of it entered mathematics. The problem then, was that far too many symbols were introduced.

Since the problems of inventing, using and interpreting symbols have always been there, alongside other developments in mathematics, confusion and mistrust among mathematicians resulted in many symbols not being standardized. Cajori (1993) describes the problems associated with this historical development and points out that, “The terms and notation varied a great deal; many symbols were derived from abbreviations. But as far as one can judge, the introduction of letters for classes of numbers was accepted as a minor move in the development of symbolism."

According to Pimm (1987), the problem is that the symbols themselves are taken as the objects of mathematics rather than the ideas and processes which they represent. Learners fail to interpret or understand the meaning of certain mathematical symbols due to the way by which they are taught to read those symbols. Learners studying alone at home usually do not know how to read many mathematical symbols because they seldom hear them being spoken. For example, some sixth form learners could read the first element of a matrix a11 as "a eleven" or "a subscript eleven" instead of "a one one" (Rubenstein & Thompson, 2001).

Researchers have found that students have preconceived ideas from personal experiences about what Mathematics symbols are supposed to represent, and often base their interpretations on these experiences, falsely assuming that all symbol use is related (Stacey & MacGregor, 1997). Lecturers and researchers of mathematics education have observed that many difficulties in mathematics can be attributed to students’ problems with manipulating and understanding mathematical symbols (Driscoll, 1999; Gray & Tall, 1994). One reason for this difficulty that is identified in the research comes from the way in which individuals apply personal meaning to symbols. According to Kinzel (1999), mathematical notations can only be thought of as potential representations that do not become representations until someone constructs an interpretation for them. One person’s interpretation may differ from another’s. Students’ own interpretations are based in the prior experiences that they bring to the classroom. As Stacy and MacGregor (1999) point out, students already have their own ideas about the uses of letters and symbols in their world, and their prior experiences often hinder understanding of mathematical language and notation (Garagae, 2011). Kirshner and Awtry (2004) give evidence that students working with mathematical expressions often respond spontaneously to familiarity with notational patterns when making decisions instead of relying on mathematical rules. Students often do not reason about an overall goal or the concepts involved in a problem, but instead look for an implied procedure inherent in the symbols.

A lot of research on how the use of mathematical symbols influence understanding of concepts has been carried out at the primary school and probably very little at secondary school or higher education institutions (Luna & Fuscablo, 2002). Yet, mathematical symbols are introduced continuously at all levels of education. However, the general consensus is that the introduction of mathematical symbols presents difficulties and challenges beyond those presented by words alone (Kuster 2010, Lee 2004).

Theoretical Framework

In order to look in depth at students' understanding of the symbolic language of mathematics, a theoretical framework is used as a lens for analysis of the data. The theoretical framework used in this study involves a subset of Arcavi's (1994) symbol sense constructed by Pierce and Stacey (2001, 2004) called Mathematical Insight. It is a lens that can be used to identify specific instances of symbol sense, particularly at the solution stage of problem solving. The authors divide mathematical insight into two parts: a) mathematical expectation is the insight needed for working within a symbolic expression, and b) linking representations is the insight needed to make connections between symbolic and graphic forms or symbolic and numeric forms. Incorporating the instances of mathematical insight is important because the stages are often mental processes that are impossible to see directly. The researcher cannot know what the students are thinking or understanding, but can try to recognize instances of symbol sense from their actions and shared thoughts and use this information to further understand their goals, activity choices, and reflections.

RESEARCH METHODOLOGY

Research approach

I used qualitative research design for exploring students‟ perceptions about the use of symbols, letters and sings in Mathematics. I preferred qualitative design because in this design because I was able to explore the understanding, use and perceived influence through words and action (Fraenkel & Wallen, 2003). In this study, I administered a questionnaire to the research participants in order to collect data in their natural setting without controlling any aspect of the research situation. This research study was intending to find out students‟ perceptions, the effect of that perception on their learning and exploring the reasons of their perceptions. These questions, which are concerned with the process of phenomenon, are best answered through qualitative paradigm (Creswell, 2003).

Research Design

The descriptive survey methodology was used to find out students perception of how symbols affect mathematics learning. However, descriptive surveys are not very informative research designs because, "Descriptive surveys basically inquire into the status quo; they attempt to measure what exists without questioning why it exists" (Ary, Jacobs & Razavieh 1985, p. 337). To overcome this shortfall, interviews in the form of focus group discussions or "oral tests" with students were held. Students were allowed to discuss other difficulties they encountered in using mathematics symbols in an atmosphere of freedom of expression and one that would ensure that they were also free to criticize their teachers or their textbooks and report back the main points (Österholm, 2008).

Sample and Sampling Procedure

Nkhoma University was conveniently chosen simply because this is where I teach and that is where the problems associated with mathematical symbols had been experienced. It is assumed that similar problems exist in other university colleges as well.
Then, all students taking mathematics course as part of their programme using the purposive non random sampling technique. The mathematics classes were chosen because the researcher wanted to use a large sample at the same time spend more time on discussions and diagnostic tests rather than on questionnaire administration only. The respondents were undergraduate students studying for bachelor of education in mathematical sciences education and bachelor of business education and the assumption was that each level had learnt set theory. This assumption was later verified to be a true statement. Since the study was about students' understanding of mathematics concepts and use of mathematics symbols, an unbiased study sample had to be chosen by controlling for the ability factor. Thus, students' scores in a tests on the topics from which the symbols were drawn.

Instruments and Data Collection Procedure

Questionnaires were given to the undergraduate students to complete while being supervised by the researcher. There were 22 questions involving interpretation and use of symbols, perceptions of use of symbols in mathematics learning. The reason for focusing on sets was that, basically, all mathematical structures can be explained in theoretic terms and the students who had been taught by the researcher before had difficulties in grasping and using the symbols.
The diagnostic tests were enthusiastically carried out by students and they participated lively. The data collected from the questionnaires revealed how students viewed the use of symbols in mathematics learning. Unfortunately, the researcher did not get quite a lot of information from the teachers themselves, most of whom gave excuses of being busy.

Analysis of data

Upon completion of the data gathering, descriptive statistical analyses were performed in order to measure the frequency, mean and standard deviation of ratings for each survey item. For the questions requiring yes or no, frquencies of yes or no were computed. Students' individual scores on mathematics digonostic questions were computed to establish the students level of performance.
The test scores on the symbols are dependent variables, and as such, are used to determine how the meaning, understanding and perceptions and use of mathematical symbols influence mathematics learning. Finally, simple statistics were conducted to determine if mathematical symbols affect students' performance in mathematics.

RESULTS

With this study, the researcher hopes to provide some answer to the question: what are the students perceptions of the use of symbols in mathematics learning? In this paper, the findings are organized around the three components of this research question.

Demographic information

Questions 1 to 3 were about demographic information. The survey involved a total of 292 university students studying mathematics as part of their undergraduate degree programmes. The sample consisted of 200 male students representing 68.5% of the sample and 92 female students. The majority of students were in the age range between 18 and 25 (91.7%). They perceived their ability in mathematics as average (71.4%) with about 27.4% as excellent.

Understanding mathematical symbols

Question 4 was, "Have you had difficulties in reading and pronouncing mathematical symbols?" The results show that the majority of students indicated that they had few to very few problems in identifying (38.7%), reading (54.8%), writing (61.7%) and pronouncing (50%) mathematical symbols. However, the majority had quite a few problems with identifying, reading and pronouncing more than writing the symbols.

In item 5 was about defining set symbols. For many students (63%), the easiest way of defining a set is by listing all the elements in that set. However, a few students (32.9%) indicated that they would define as set by describing the elements. These results indicate that the majority of students (about 66.7%) had difficulties in reading and pronouncing mathematical symbols.

Question 6 was on what could be the easiest way of defining a set. Students were provided with the alternative responses to respond to and the frequencies were calculated. The results show that, By listing its elements 73.3%(88); By describing the elements 26.7%(32) and Any other way (specify) 0%(0). Here language problems are revealed because one can conclude that most students wanted to "list" and not to "describe" symbols.

In question 7, there were only two alternative responses, A:YES or B:NO, to the given statement that many students find sets difficult to understand because some symbols which are used look alike but mean different things. The responses were 61.7% (74) for A and 38.3% (46) for B. Thus the majority agreed with the statement and this tallies with the researcher's assumptions and also with findings of Rubenstein and Thompson (2001).

Use of symbols in mathematics learning

Item 8 was about the difficulties that students had in using symbols in sets. Interestingly, there were two (more or less equal) groups with contrasting views. Of all the students, 43.3% indicated that they had quite a lot of difficulties whereas 20% indicated that they had very few difficulties. The frequency for each item was computed. Hence, it seems that on average, students had problems in using symbols in sets.

In question 9, symbols for the improper subset, less than or equal to, element of, and universal set were given and students were told that the symbols looked alike. Of the students, 1.7% (2) strongly agreed, 1.7 % (2) agreed, 21.6 % (26) were undecided, 41.7% (50) disagreed and 33.3% (40) strongly disagreed with the given statement. This might mean that students were quite familiar with the "structure" of the symbols. However, this may not mean that they knew the meanings of those symbols.

Question 10 tested students on the distinction between "subset of" and "contains". The question was: - Does the symbols used to connect A and B as in Does

$A \subset B$ *and* $B \supset A$ mean (a) exactly the same (b) almost the same or (c) opposites? Here 73.3% (88) chose (a), 20% (24) chose (b) and 6.7% (8) chose (c). The results indicate that students couldn't distinguish between $A \subset B$ *and* $B \supset A$

In item 11, the following symbols were given: symbols: $\cup, \subset, \cap, \supset$, and students were told that it is just the same symbol rotated four times through an angle of 90o clockwise; so it stands for the same thing. The results were that 5% (6) strongly agreed, 5% (6) agreed, 26.7% (32) were undecided, 43.3% (52) disagreed and 20% (24) strongly disagreed. It can be concluded that there are students (10%) who still think that a symbol must retain its meaning even if it is rotated in such a way that it looks different from its original structure. They probably do not know that rotation (or orientation) changes the meaning of the symbol completely.

In question 12, the fact that the majority of students (50.7%) described as the opposite of each shows their ability to identify symbols correctly. However, the 31.5% of students who indicated that the two expressions were exactly the same can be worrisome There is a thin difference in percentage of students who indicated that the symbol used to connect A and B looked a like. In fact, they were different. This perception augurs very well with their perception of the majority of students (57.5%) that sets are difficult because symbols used look alike but mean different things. Furthermore, about half of the students involved in this study (50.7%) indicated that they had difficulties in using connecting the symbols with the concepts to be learnt

Question 13 required students to name some topics in mathematics where they have had difficulties in understanding the symbols given. It was observed that sets, probability, inequalities, variation and trigonometrical ratios were some of the topics mentioned by the majority (93.3%) of the students. This confirms the researcher's hypothesis that most students have problems understanding mathematical symbols and these problems might reduce performance in mathematical problem solving (Luna & Fuscablo, 2002).

The symbols E and ξ representing the entire set were given in question 14 and students were asked to choose the symbol they preferred to use and to state why. Fifty five percent (66) chose E and 45% (54) chose the latter symbol. Those who chose E gave reasons that it was "easy to write," or "more understandable" and that "the other one is complicated". Those who preferred the latter symbol gave reasons that E could not be a better one since it means "an element of," thus confusing it with Î.

Question 15 asked for those particular symbols. Some of the mentioned ones were symbols for less than or equal to, subset and "contains", union and intersection, alpha and the universal set. Since these symbols have to be explained in English, students may also have a "double jeopardy" if English is not their first language (Garegae, 2011).

The reasons for not being able to grasp the meanings of the symbols and to use them appropriately were indicated in question 16. Thirty-three percent (40) of the students blamed the lecturers who "didn't explain what the symbols meant", 5% (6) gave various reasons such as "no enough time to study", "no skilled lecturers" and "there are too many symbols in mathematics" while 62% (74) blamed the "shallow textbooks" and themselves for their failure to use mathematics symbols appropriately.

The meaning of n(E) was tested in question 17. There is actually a difference between a(x) which means a times x in mathematics and n(E) which does not mean n times E in sets. So, for the statement that n(E) means n times E, 28.3%(34) chose A:TRUE, 65%(78) chose B:FALSE and 6.7%(8) of the students chose

C:UNDECIDED. From these results one can conclude that most students understood the meaning of n(E).

In question 18, the main idea was on using different symbols to represent an empty set. There were three alternatives, namely, (a) an empty circle (b) B= { } and (c) B=Æ. It is interesting to note that 66.7% (80) of the students thought that response (c) was the best to stand for "set B is empty" while 33.3% (40) chose B={ } and 0% (0) preferred just an empty circle with a letter outside the circumference.

Student use of symbols in mathematics learning

In question 19, a circle, B, was drawn and inside it were two elements, a small circle (or zero) and the letter A outside it but near the circumference. There were four alternatives as to what the diagram might represent or mean, namely, (a) Set B contains two elements; the letter A and the number 0 (b) Set A is a subset of set B (c) Set B contains two elements; the letter A and a small circle (d) Any other conclusion (specify). All the responses a, b, and c would be correct. Thirty five percent (42) chose (a), 18.33% (22) chose (b) and 45.83% (55) chose (c). Only 0.83% (1) chose (d) but did not give any reasons. Maybe he or she did not know the meaning of "specify".

Question 20 included a universal set E with two intersecting subsets A and B. There were some geometric shapes as elements in each set. Students were asked to describe fully and in words the set E. Language difficulties in mathematics were revealed in the responses to this question. About 78.3% (94) of the students could not describe the given set E fully and correctly. Those who did better, 21.7% (26), did so but still in poor English.

Question 21 was If $n(A)=250, n(B)=200\, and\, n(A\cap B)=80$, find $n(A\cup B)$. The correct answer was 370. However, some students did not apply the formula correctly by adding the two sets only or adding the two sets and got 450 and also adding the intersection of the two sets and got 530 (16). It was also observed that some students (8) did not attempt to answer this question suggesting that they had no clue as to how to solve the problem.

Student perceptions of use of symbols in mathematics learning

Item 22 was the statement that students fail mathematics because there are too many symbols to learn and understand. The respective responses and frequencies were as follows:-

Degree of agreement	Frequency	Percentage
Strongly agree	52	18
Agree	102	35
Undecided	36	12
Disagree	58	20
Strongly disagree	44	15

These results indicate that most students agreed that failure of mathematics is caused by too many symbols which need to be learnt and understood. The implication is that symbols are perceived as a distractor to effective learning of mathematics.

When asked whether students fail mathematics because of failure to comprehend the symbols used, the majority indicated that this is very possible (58.9%). When presented with the majority indicated that these symbols looked alike though they meant different things. Furthermore, students were almost divided on whether the symbol used to connect A and B looked alike. To the majority of students, n(E) did not mean n times E (56.2%) although38.4% of the students indicated otherwise. The majority of students indicated that ϕ represent an empty set whereas 30.1% indicated that (o) is the correct representation of an empty set

DISCUSSION

Symbols as part of mathematical language

From the questionnaires that were administered and the interviews and diagnostic tests carried out with students, it was found that some difficulties in learning mathematics arise not from the vocabulary of mathematical writings but from their linguistic "structure" (Miura, 2001). Thus, language plays a key role in the use and mastery of mathematical symbols. Pimm (1987, p.148) has similar views and further suggests that, "It clearly depends on the language of the reader, and on occasion,...,there will be a conflict between the conventional letter employed in Mathematics and an unwillingness (on grounds of easing the memory load) to employ that particular letter."

Most students responded to questions in the questionnaire by accepting that there were many symbols in mathematics some of which varied a lot in meaning and/or structure. This is in line with Pimm (1987) who criticises the great variation in symbols and terminology used in mathematics. But there are situations in which expressing mathematical ideas in more than one way may be beneficial. Pimm (1995, p. 287) writes, "Situations in which the use of more than one system of notation may be either helpful or confusing, for example, could be quoted." Unfortunately, the author does not quote those situations.

Mathematics is a language used to express mathematical relationships. Students need to understand how mathematical concepts are related to one another and how symbols can be used to concisely express and analyze those relationships. The aim of the study was to explore students‟ perceptions about the use of symbols, letters and signs and the effect of their perceptions on their learning of Mathematics. The study revealed that the students have many misconceptions in the use of symbols in Mathematics, which affect their learning of Mathematics.

Functional mathematical symbols

It is vital that students recognize that the symbols that are used to represent an unknown quantity or variable have different meanings in different contexts. Mathematical symbols are so significant as a part of Mathematics that its foundation must begin to be built in the very early grades. It must be a part of an entire curriculum which involves creating, representing, and using symbols for relationships. But getting desired objectives teachers‟ content knowledge and content provided by textbooks also play a significant role for promoting students relational knowledge and conceptual understanding of Mathematics (Österholm, 2008). For relational understanding the concepts of Mathematics and use of Mathematics as a tool to use it in real world situations it is important that the teachers should develop students‟ mathematical thinking and symbol sense. To assure that all children have conceptual understating of the use of symbols in Mathematics, these concepts must be incorporated throughout the entire Mathematics curriculum. So, that all students could know and apply Mathematics in solving their real problems confidently regardless of their ultimate career.

Students perception of use of symbols in mathematics learning

This study has also investigated how students perceive the use of symbols in mathematics learning. From the findings of the study, there is evidence that most students fail to interpret or understand the meaning of mathematical symbols due to the way by which they are taught to read, pronounce and use them. This misuse (and abuse) of symbols considerably hinder formation, understanding and communication of concepts to a great deal and might affect the final achievement.

Introducing mathematical symbols to students

This study has also concluded that students fail to grasp mathematical concepts because they take the symbols themselves as the objects of mathematics rather than the ideas and processes which they represent. According to the results from the questionnaires, the blame lies on the textbooks and the teachers. Teachers seldom explain the meanings and proper uses of the symbols while textbooks change the symbols too often and don't bother to give historical background information about those symbols (Österholm, 2008). Students fail because teachers introduce new words or symbols when the given situation can be handled in terms of words and symbols already known. Drawing from the problems and difficulties mentioned above and their possible causes, the following recommendations for an effective and proper use of symbols that would lead to a firmer grasp of mathematical concepts are given.

On textbooks, it is the duty of everybody concerned with mathematics education to improve the text, the teacher's use of the text and the reading ability of the reader (Österholm, 2008). When recommending textbooks, teachers should select those that provide short historical accounts of mathematical symbols they used. The textbooks should explain why certain symbols were dropped and yet others were accepted internationally. For exampple, it is not explained why „x“ is dropped in favour of „.“ . If such textbooks are not available, historians, mathematicians and educators can work together to produce them. The teachers and textbooks should avoid continuous use of symbols that are complicated and difficult to understand, difficult to write (sometimes needing a computer) and confusing and contradictory. It is helpful for teachers to make sure that students understand the meanings of the symbols even though they allow the students to manipulate such symbols mechanically.

The first lesson about symbols should emphasize strongly the fact that symbols are instruments or tools of thought. Another lesson should focus on the fact that a given symbol may often serve a variety of purposes. For example, the symbol "e" is used as a base in logarithms, as the identity element in abstract mathematics and as the coefficient of restitution in mechanics. Thus, it is important to study the setting and context in which the symbol is used.

The teacher should also be well versed in mathematics in general and in the use of mathematicss symbols in particular. In the classroom the good teacher can introduce games involving the use of symbols, constantly referring to the school library section on mathematicss games. Using overhead or micro-soft power point slide projectors, any chosen student can read aloud the written symbol, tells the topic (area) where the symbol is used and then spells the "name" of the symbol while others record time. These activities could be done at all levels of education as remedial work for slow learners. The teacher should also display on the wall several charts which carry different symbols, what they mean and where they are used. Students can then practise pronouncing them and using them in sentences as suggested by Rubenstein and Thompson (2001).

If symbols interfere with the smooth learning of mathematics, then it is true that symbols can be weapons of math destruction. Asking students to work through a mathematics problem that involves unknown symbols is like asking a child to play with a bomb; once it explodes, the students gives up mathematics learning altogether. Thus, mathematical symbolism should be integrated with other topics or subjects at the beginning of every course and be sustained at all levels of students' learning (Luna & Fuscablo, 2002). Teachers should not take symbols for granted and should not by-pass them in their discussions (Arcavi, 1994).

CONCLUSION

In conclusion, symbols should be used only after a satisfactory explanation of their meanings has been given, otherwise they should be accepted worldwide. Errors in reading and pronouncing symbols should be identified and remedied. The meaning of each symbol or each symbol string should be razor sharp and unambiguous. That way, mathematical concepts can be firmly understood and grasped. Teachers are hereby challenged to use suggestions and recommendations given in this study and to carry out classroom action research on the use of symbols and their impact on student achievement.

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Educational neuroscience and the critical role it could play in mathematics teacher education curricula

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This paper evolve around current research in educational neuroscience and the role it could play in initial mathematics teacher education curricula. Mathematics instruction and knowledge acquisition is embedded in mathematical cognition which is the cognitive functioning of the brain as it engages in mathematical interactions. Mathematical cognition is the mental process and neurological spell involved in mathematical knowledge acquisition and refers to thinking, understanding and remembering and the conscious mental activity conceivable in achieving aspects of awareness, perception, reasoning and judgement. The proponents of mathematics cognition have related the subject to neuroscience because cognition occurs in the brain through neurological interactions. By definition neuroscience is the science that relays the physiology, biochemistry and molecular biology of the brain, its nerves and nerve tissues in their relation to behaviour and learning. Educational neuroscience on the other hand is an emerging field that is a composition of cognitive neuroscience, developmental cognitive neuroscience, educational psychology, educational technology and educational theory. This presentation is a review of how the knowledge of educational neuroscience by teachers of mathematics and its inclusion in mathematics teacher education curricula can enhance our understanding of how students learn mathematics. The presentation is premised on studies that point to two of the obstacles that teachers of mathematics face, that is we know almost nothing about how people do mathematics and we almost know nothing about how people learn how to do mathematics. Teachers' understanding and knowledge of what goes on in the brain and mind as learners grapple with mathematical concepts and facts could enhance the way we teach and disseminate mathematical information.

BASIC NEUROSCIENCE CRITICAL FOR TEACHERS OF MATHEMATICS

Development in the interest of in the application of neuroscientific discoveries to educational practice go back four decades ago (Sousa 2010). In 1983, Leslie Hurt in her now classic works *Human Brain and Human Learning*, wrote *'teaching without* *awareness of how the brain learns is like designing a glove with no sense of how the hand looks like' (p.13).* This statement implies that teachers' understanding of how children grapple and understand mathematical concepts is critical in ensuring effective mathematical knowledge acquisition. Almost in the same vain Ansari (2010:128) posted that *I would contend that the most effective way of bringing neuroscience into the classroom is to provide teachers with access to knowledge that neuroscientific studies are yielding. This knowledge will inform teachers' conceptualization of the learning . . . And therefore their pedagogical approaches*. These are very important quotes that point to the rationale of teachers' knowledge of basic neuroscience. Advocates of educational neuroscience (Verschaffel1, Lehtinen & Van Dooren 2016; Sousa 2010 Stern & Schneider, 2010) have however hinted at some of the skepticism that neuroscientists hold about teachers being exposed to neuroscience, however rudimental it maybe. This paper advances that educational neuroscience, especially the science of learning and memory would be pivotal in teacher education curricular because of the various outlined benefits such knowledge would bring to the teachers.

Basic brain neuroanatomy

Developments in imaging technology have propelled development in cognitive psychology and neuroscience. Before then cognitive scientists drew conclusions about brain growth or development by watching how the subjects acquired certain skills, neuroscientists could only infer about brain functions by looking at case studies from patient traumas, strokes and lesions of haemorrhage. The brain could only be studies in an autopsy. The information collected could only inform neuroscientist about where in the brain structures something happened but not the function of the brain. Machines such as the X-rays only revealed hard tissue such as bones and also damaged health brain cells. The Computerised Axial Tomography CAT or CT scan came into use in the 1970s, it had lower X-rays and was able to show variations in soft body tissues. The major breakthrough in medical diagnosis of the brain came with the use of the Magnet Resonance Imaging in the 1980s. These were great for medical diagnosis of the brain traumas by showing the structures that were affected, but what the scientists needed most was technology that would reveal the function of the brain. The functional Magnet Resonance Imaging (fMRI) was the answer.

The discovery in the 1970 to 1980 about the brain being made up of various regions that functioned independently formed the basis for explaining why different learners have different learning styles and that began the movement to link pedagogy to neuroscientific discoveries (Sousa, 2010). Educational implications in neuroscience and mathematics attest that teachers of mathematics could benefit from knowledge of brain and its basic circuitry (Sousa, 2010; Verschaffel, Lehtinen & Van Dooren, 2016). The brain is part of the Central Nervous System. The brain as we know it today has been explained to the best of the current scientific explorations but we would be naïve to believe that we have arrived and established all the functions of the brain and how it operates. There are currently more than 10 trillion known connections between neurons in the human brain that can produce varied behavioural capabilities in a human being (Taylor 2010:48). That means there is still a lot to learn about the brain. In a learner's attempt to acquire a mathematics concept, there are several parts of the brain that are called into action. The three main parts of the brain cerebrum, the cerebellum and the brain stem are all in one form or another involved when mathematical information is relayed to the brain (Purves, Augustine, Fitzpatrick, Hall, La Mantia & White, 2012). The cerebrum is divided into two parts the right and left hemispheres. The four lobes, Frontal, Parietal, Temporal and Occipital are part of the cerebrum. Figure 1 is an attempt to isolate the core parts of the brain that a teacher of mathematics should be aware of when providing instruction. The thalamus is the part of the brain that relays information from the sensory organs (eyes, ears, skin, tongue and nose) through the sensory neuron to the cerebral cortex which is responsible for complex thought processing such as mathematical cognition (Purves et al. (2012). The brain communicates with the support of neurons or nerve cells as they are sometimes referred to.

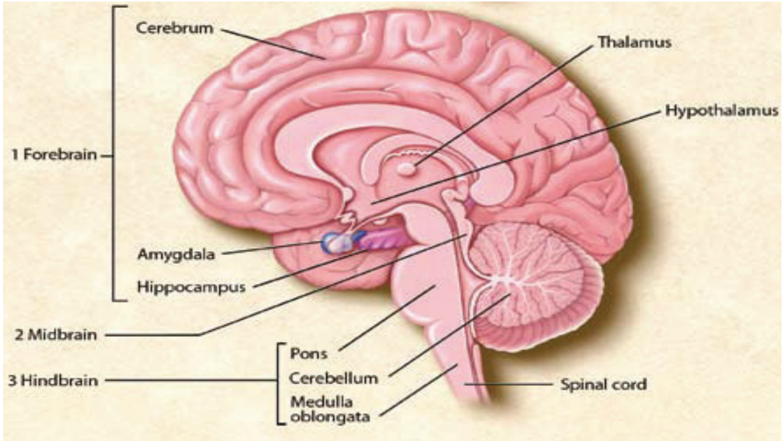

Figure 1: Image from *Brain Facts: A Primer of the Brain and the Nervous Sysem*, p.5. 2008 Society for Neuroscience

The neuron and its doctrine

The neuron is a cell that is made up of the nucleus, cell body the Selma, dendrites and the axon. The neuron doctrine was expounded by a Spanish neuroanatomist by the name of Santiago Ramon Cajal (1852-1934). He used the Golgi staining technique to individualise the cells and pointed out that cells have each got a separate morphology and not a continuous process or system as was earlier defined by Camillo Gogil an Italian neuroanatomist with the reticular theory that advocated that cells morphology were continuous. Cajal using Golgi staining method expanded on the structural molecular uniqueness of neurons and their connectivity with other cells via the synapse (Poo, 2011).

Most of communication in the brain is transferred from one neuron to the other as an electro chemical impulse called action potential. As earlier stated action potential is the signal by which cells (neurons) communicate in the body. The brain has 100 billion neuron and no one knows the number of connections between them in the nervous system. There are two types of cells in the body. Neurons are electrochemical producers and transmitters and support cells such as glia cells that guard and insulate neurons. The signal in the neuron is intra cellular and passes through the axon to the neuron terminal. The electrochemical signal is due to the movement of ions which are as a result of change in potential in the cell membrane called polarisation. The change in the membrane is in response to an external stimuli such as a mathematical input. The signal as an action potential is measured in millivolts. The action potential is sometimes referred to as the propagation of a change in the cell membrane. Neurons are said to be electrically excitable. The potential difference in the cell membrane is due to an influx of sodium ions. When the signal or the action potential reaches the synapse, it creates a synaptic potential that is the depolarisation of the synapse to enable a transmission of the chemical signal between the two cells. The polarisation or depolarising potential is due to the activation of neurotransmitter receptors at synapse. The Excitatory Postsynaptic Potential (EPSP) or current occurs in the receiving neuron and is the measure of the synaptic strength at excitatory synapse.

The axon as the neuronal path allows rapid transmission of the intracellular signal due to a movement of irons, the action potential as the cells have a potential difference – membrane potential more positive ions outside (lots of sodium ions, potassium) inside cell membrane more negative – low sodium and higher potassium. The transmission of a unidirectional signal is the goal of imbalance in the cell membrane and each depolarisation is an action potential

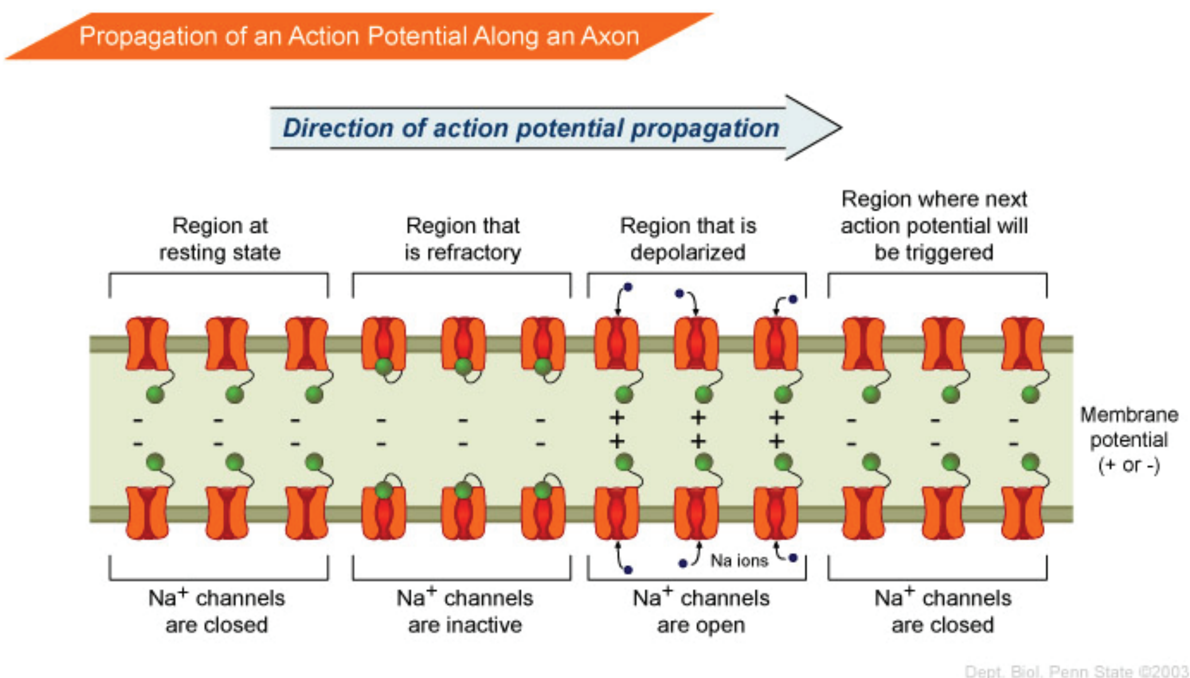

Figure 2: Propagating of Action Potential along the axon (Adopted from Department of Biology Penn State University 2003)

Figure 2 shows that during an action potential which occurs within milliseconds, the voltage-gated ion channels open and allow an influx of sodium ions and that creates an imbalance in the internal part of the membrane of the neuron. After the potentiation the channel gates close and the neuron is at resting- equilibrium potential. The equilibrium state is acerbated by potassium ions that are channelled outside the neuron and that creates an increase in positive ions outside and an accumulation of negative ions inside the neuron.

Regarding an action potential as a mathematical signal

When mathematical instruction such the provision of a concept definition is provided to the learner, the message is relayed through the brain cells as an action potential but not every action potential results in learning. The signal travels the axon to the nerve end of the pre synaptic cells also known as presynaptic neuron. At the presynaptic cell end the action potential potentiates the release of presynaptic neurotransmitters at the presynaptic region of the neuron cell terminal through vesicles. Vesicles carry the neurotransmitters and if the presynaptic cells are in excitatory mode and the post synaptic cells are also in the excitatory mode (excitatory synapse) the vesicles releases the neurotransmitters as chemical reactions into the region between the transmitting neuron and the receiving postsynaptic neurons called synapse. This results in synaptic polarization or depolarizing of the postsynaptic neuron. If the excitatory neurotransmitters are strong enough to reach the threshold of excitation, then the neuron will fire – an action potential that will relay the mathematical impulse further down the neuron. Through a process called reuptake the empty vesicles without neuro transmitters go back into the cell axon to start the process all over again. Figure 3 provides an illustrative explanation of the action potential at synapse that enables the release of neuron transmitters and attach to the receptors of the post synaptic neuron.

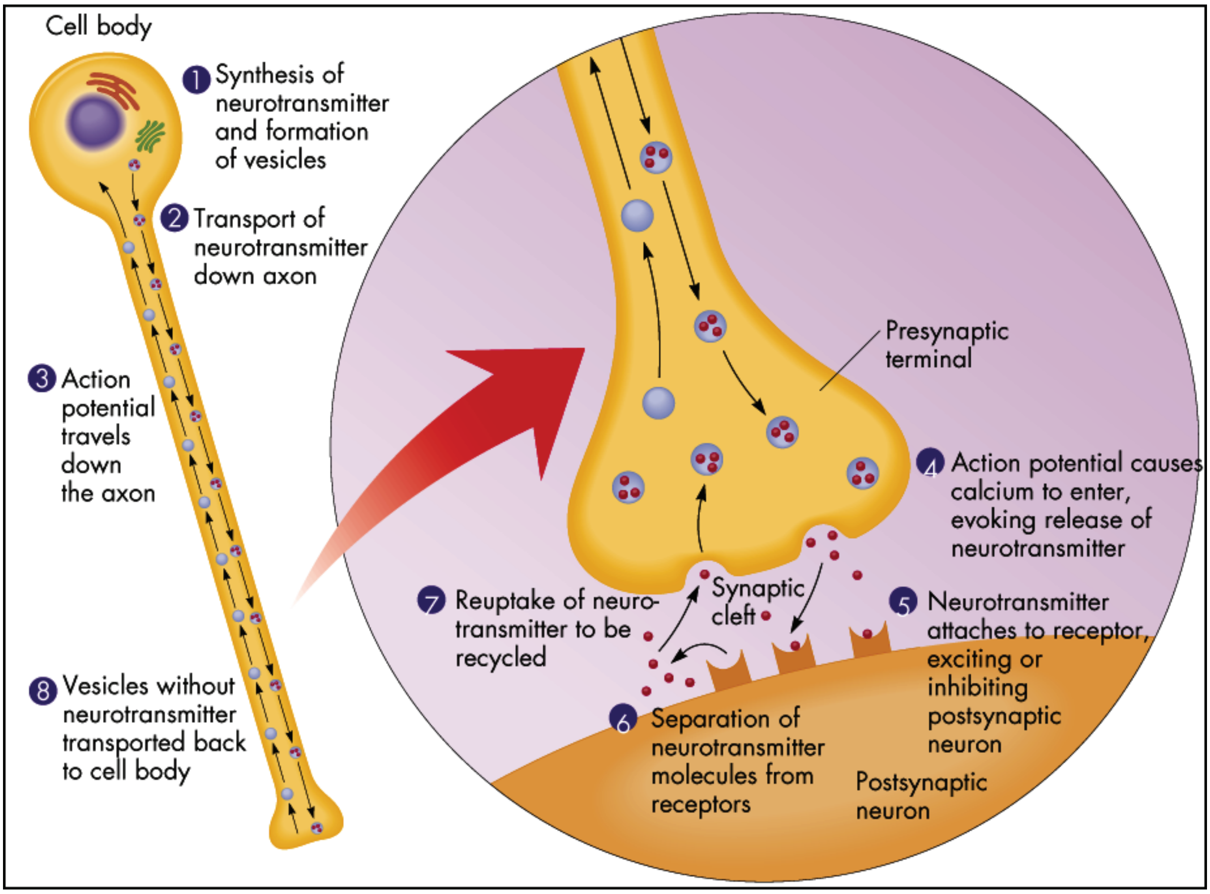

Figure 3: Adopted from Neuron Synapse Anthropology.net Action potential at Synapse and role of neurotransmitters and receptors

Transmitted Neuronal Signal

Nerve cell communicate by the action potential and a means of transmitting information which is the unit of communication in the nervous system. Transits signal through a potential changes - action potential – nerve impulse (polarisation or depolarising), it is a large but brief depolarisation of the membrane potential that occurs within millisecond. The action potential which is the unit of signal transmitted by neurons occurs by depolarisation or hyperpolarising the membrane potential at synapse and fires from -70 mV to + 40 mV, this occurs within milliseconds. The figure below shows an example of an action potential above threshold.

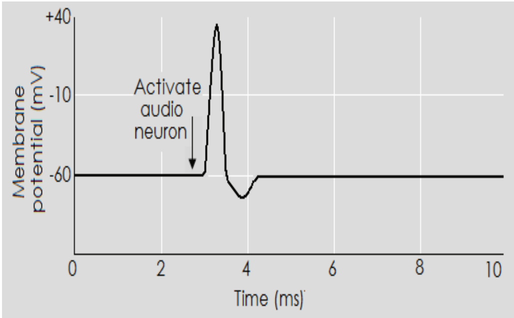

Figure 4: *Graphic representation of an action potential of an audio input above threshold.*

Only when the depolarisation is above threshold is there an action potential that transmits as a result of a synaptic potential. Figure 5 below show the various stages of mathematical cognition from the introduction of a mathematical stimuli neurological synaptic modification as a trace left at the synapse and encoded as memory to change in behaviour. The change in behaviour can be strong if the input is continuous at specific group of cells and conveys perceptual memory. A weak cell assembly yields weak cognition and the result are concepts that are not well understood and stored in short term memory and not easily recollected for reuse in other mathematical situations.

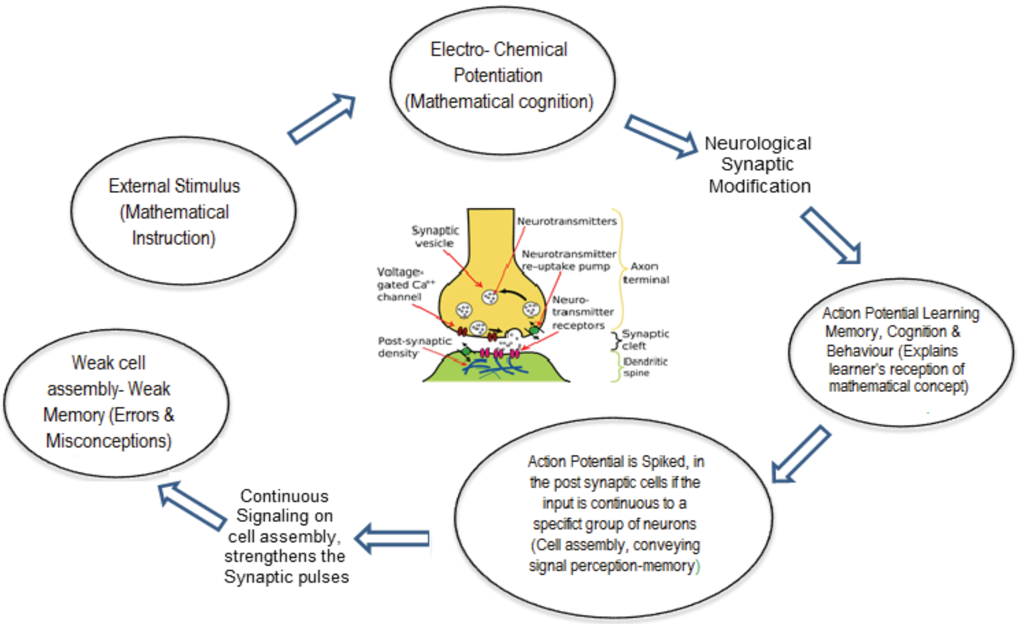

Figure: 5 *Partially adopted from Bear et al 2001: Illustrations of cell assembly with additions from Synapse Illustration.jpg*

The cellular basis of mathematical cognition

As mathematical cognition is the mental process and neurological engagement involved in mathematical knowledge acquisition. Mathematical cognition refers to thinking, understanding and remembering and it is the conscious mental activity conceivable in achieving aspects of awareness, perception, reasoning and judgement. Mathematical cognition is a transmitted neuronal signal. As a mathematics teachers explains a concept a mathematical signal is sent to the recipient, the learner. The signal causes a depolarisation in the neuron that leads to an action potential which is the signal unit of the reaction. Every mathematical explanation causes a depolarisation and an action potential.

At the synapse the synaptic potential which is a polarisation or hyperpolarising potential due to a reaction at the synapse due to a large depolarisation of the cell membrane from about -70 mV to + 40 mV into an action potential and occurs with milliseconds. The neuron fires when depolarisation reaches threshold.

The transmission of the mathematical signal to the next cell/neuron is due to the synaptic potential. When a group of synapses or mathematical impulses work together they causes an Excitatory Post Synaptic Potential in the receiving neuron. The information is gathered from different inputs to form the excitatory post synaptic potential in the receiving neuron. The persistence and continuous bombardment of the post synaptic cell similar mathematical impulses leaves an imprint at the synapse and that is mathematical concept acquisition and memory.

When children do not understand mathematics

In cognitive neuroscience Hebb (1949:136) postulate that "When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased" is pivotal to teachers' mathematical instruction, context of instruction and learners knowledge. The Hebb's learning Rule- 'Cells that fire together wire together' implied that Correlated pre and post synaptic activities cause synapse to strengthen/stabilisation. In explaining mathematical learning none neuroscience studies have emphasised the importance of relating mathematical content to what the learners already know, prior knowledge as that strengthens the new mathematics and mathematics concepts would be understood. Uncorrelated pre and post synaptic activities cause synapse weakening or even elimination leading to mathematical concepts not being understood or concepts being quite easily forgotten. The converse is that when the mathematical stimuli causes an action potential, the synaptic connection is strengthen when Cell A keeps firing Cell B, correlated pre and postsynaptic mathematical impulse causes synaptic stabilisation or strengthen the understanding of the mathematical concepts. The postsynaptic cell is an integrator of all the pre synaptic signals or mathematical impulses and a bundle of uncorrelated mathematical signals will yield uncorrelated mathematical outcomes (Poo, 2011).

The Long Term Potentiation (LTP) and mathematical cognition

Bliss and Lomo (1973) in a now classic paper in most memory and learning studies explained in detail how similar sets of mathematical impulses or neurological stimuli that leads to understanding and memory is traced to the various regions of the hippocampus in brain cortex. In their study on the brain of a rat they discovered that a high frequency stimulation of this region of the cortex a synaptic transmission is enhanced for a prolonged period and this is memory (Bliss &Lomo 1973). The frequent transmission (persistent mathematical input) induces the cellular changes in the hippocampus which can be explained as the trace of memory of the prolonged experiences of the mathematical impulse. The cellular change at the synapse in the hippocampus creates memory which is an electric long term trace of experience and in this instance mathematical experience causing perceptual learning (Poo, 2011). The LTP explains why in explaining mathematical concepts repeated experiences which are revision, sometimes re-teaching of the concepts and class and homework exercises would be important for memory and understanding. The cellular change at the synapse cause the perceptual learning over a longer period of time - memory.

How related mathematical concepts are explained as inducing LTP – Input specificity

LTP has a property that is input specific (Poo, 2011; Bears et. Al. 2001) and this provides further explanations to mathematical knowledge acquisition and cognition. The studies (Bears et al. 2001) explain that if the hippocampus neuron dendrite receives input from two different sources, the side which is highly stimulated (100 Hz/sec) would produce the synaptic amplitude of the EPSP to be higher and lasts longer (memory). The other side of the dendrite will not be potentiated. This is a synaptic modification due to a correlated firing of cell A and B according to Hebb (Poo, 2011) and only the side of the dendrite that was related to the input get potentiated. The action potential will not occur at the other side of the dendrite where the input is not correlated to the neuron. Only connection between two specifically correlated neurons will be potentiated.

Long Term Potentiation (LTP) due to Associativity

One of the property of LTP is due to neuron's association with similar input. Poo (2011) explains that if we have two neurons one with a weaker input and another with a stronger input and if the one with a weaker input is stimulated the Excitatory Post Synaptic Potential (EPSP) or current, which is a measure of synaptic strength at excitatory synapse will not be potentiated and therefore will not produce LTP. The synaptic amplitude of EPSP remains the same as can be seam below.

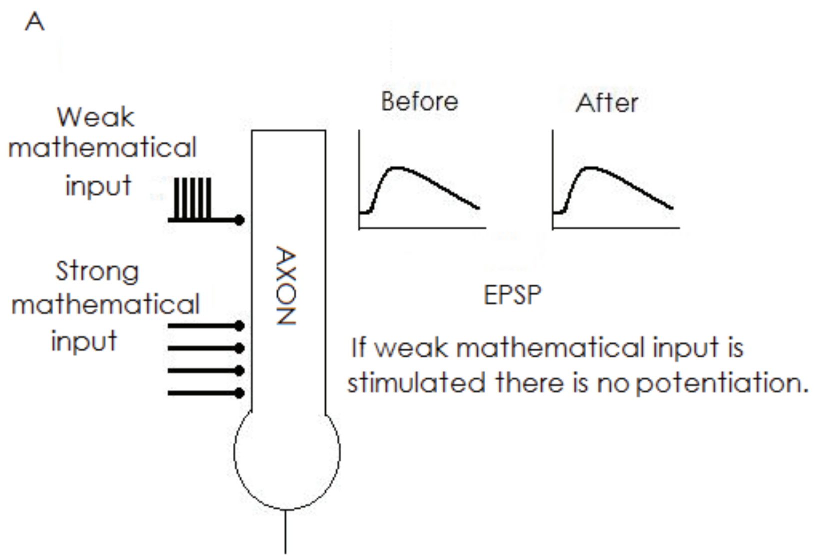

Figure 6: *Excitation of weaker mathematical input does not affect EPSP*

If the stronger input is stimulated with a high frequency, the post synaptic cells activated will be able to potentiate and activate the LTP as shown figure below.

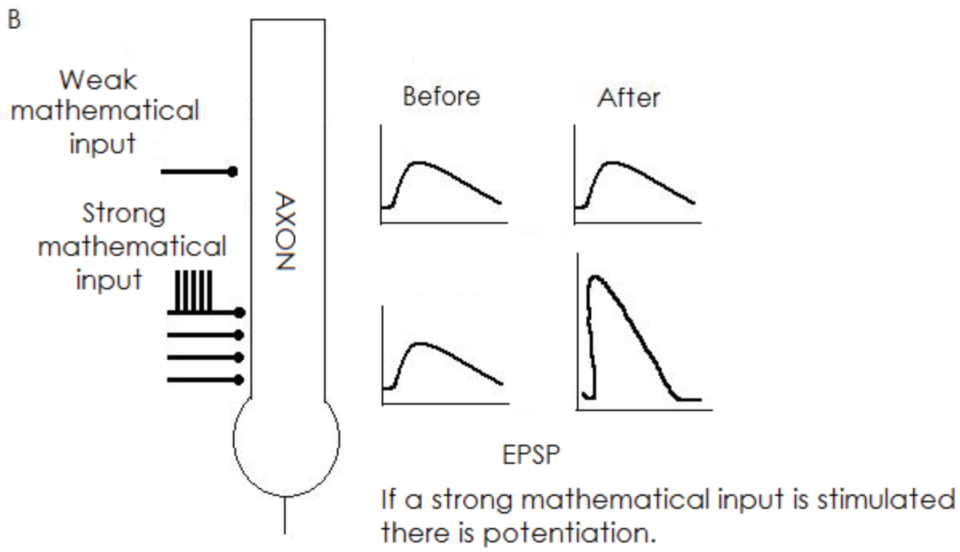

Figure 7: *Strong mathematics input is stimulated it there is action potential - high EPSP*

However if the weak input is associated with the stronger input by administering a strong frequency at the same time as the stronger input the weak input is potentiated and the LTD is stimulated in both inputs and the synaptic amplitude of EPSP goes up in both inputs.

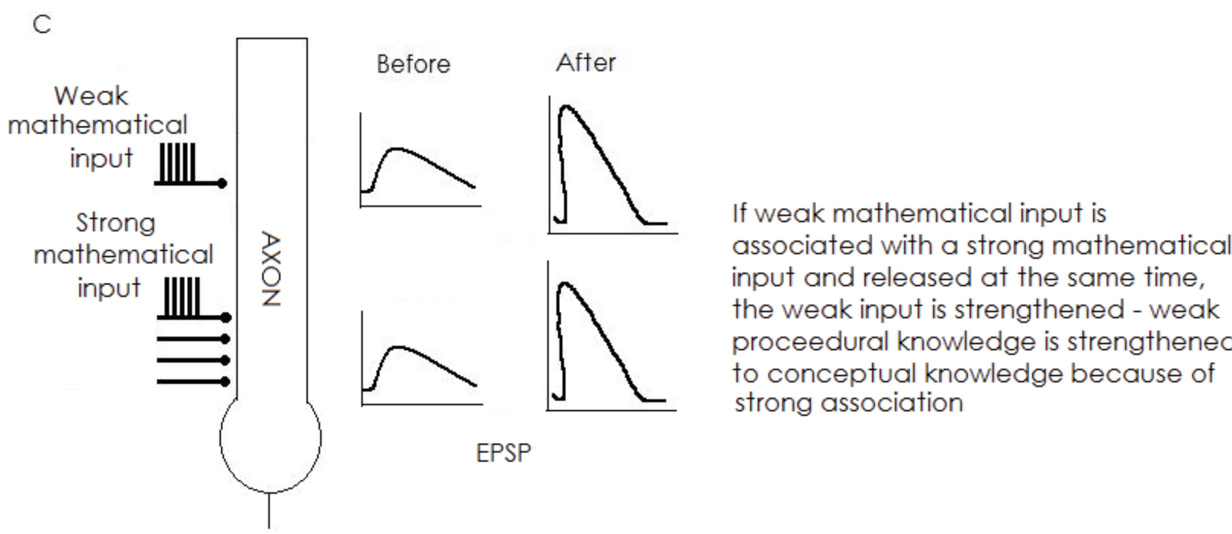

Figure 8: *Weak mathematical input strengthened by correlated stimuli associated with stronger input*

The descriptions here are critical to mathematical explanations where associativity of mathematical concepts with other related concepts is critical to conceptual understanding.

The essence of interconnectivity of input

Input such as a mathematical signal is strengthened when it is connected and relevant, hence when mathematics is being taught research (Stein at al 2006; Schoenfeld 2014) show that mathematical information should be related to what the learners know. Neurologically, synapses are strengthened by correlated activities (Cell that fire together wire together) and that perceptual memory of sensory experience involves the formation of a specific group of interconnected cells (Cell assembly) (Poo, 2011; Bear et al 2001)). Mathematical input should therefore point to information that is related, topical and targeted at specific learning outcome. When mathematical explanations – input is targeted at specific topic with examples, illustrations, class exercises and homework the learner develops strengthened LTP, the connections between the cells is strengthened and this is perceptual learning. There is therefore neurological evidence that understanding mathematics is a result of repeated association of concepts with previously learnt work

In explaining the mathematical cognition, the input of mathematics concepts will activate specific areas of the cortex respond and stored in one area. This means that the reaction or the synaptic potentiation of neurons are specific to mathematical input and input on the other side of the same dendrite that is none mathematical such as history or language will not potentate the neuron. Only specific synapses that are mathematical will modify that area of the hippocampus – different sites for different inputs. Dehaene (2011) research on the concept of 'number sense' - the symbolic representation of quantity as an important foundation for mathematics and laying in specific areas of the cortex. Cantlon et al (2006) used functional magnetic resonance imaging (fMRI), a neuroimaging technique, with adults and children to examine whether there is an early-developing neural basis for abstract numerical processing and area known as the intraparietal sulcus (IPS) was identified as corresponding to the processing of numbers.

Hussain (2012:8) expounds that 'Of relevance to educational psychologists is that some learners are characterised by specific difficulties understanding number concepts, lacking a sense of number and quantity, and have problems learning number facts and procedures, and such skills have been linked to the developing brain'. Critical here, especially to teachers at elementary school is that children's brains at this stage are still at their developmental stage and lack of effective mathematical cognition leads to dyscalculic. This is a condition that affects the ability to acquire arithmetical skills. Dyscalculia learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers and have problems learning number facts and procedures (Hussain, 2012:8). Studies such as that of Wilson et al (2001) have used current discoveries in neuroscience to develop computerised educational interventions for learners with dyscalculia. Hussain (2012) points out further that these studies used personalised instructions on the concept of number sense, for instance, to evaluate learners' performance and relate it to the difficulty of the tasks provided. Temple et al. (2003) asserts that mathematical stimuli that addresses learners conceptual enhancement and mathematical performance have neural link, which results in increase in brain activity in areas that were originally under activated.

Conclusion

There is therefore neurological evidence that understanding mathematics is a result of repeated association of concepts with previously learnt work. In order for learner to understand and consolidate conceptualisation of mathematical concepts frequency exposure to mathematical concepts has a better effect on memory than length of the exposure. Repetition and summary of covered content creates LTP.

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Teachers knowledge on students thinking towards learning mathematical concepts of area of a triangle in Primary Schools in Nairobi County.

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Abstract

This presentation is based on a larger study whose purpose was to explore the teacher's knowledge on learners thinking towards learning, area of a Triangle in primary schools in Nairobi County. The study adopted a descriptive survey design. An observation schedule were used to collect data. The study was carried out in three public primary schools in Nairobi County. Data was analyzed using descriptive statistics. Findings from the study, indicated that facilitators considered learners prior knowledge in their lessons, but they did not tackle their difficulties, mistakes and misunderstanding learners asked questions to them. They had the limited methods for constructing on learners mathematical ideas because of inadequate content knowledge.

Students and Teachers Thinking towards Mathematics

In Kenya mathematics is a compulsory subject in both primary and secondary school curriculum. It is found in volume two of the primary school syllabus. Mathematical thinking is considered one of the most important targets in Mathematics education in Kenya. Mathematical thinking can be defined as a combination of complicated processes involving conjecturing, induction, deduction, specification, generalization, analogy, reasoning, and confirmation, Tasdan, Erduran and Celik (2015). Learners and creative thinking skills can be considered to be fundamental to the learning and application of mathematics. The development of these thinking process enable learners to work mathematically and become effective problem solvers. In the problem solving process, learners think mathematically as they create and evaluate knowledge, noting possible methods, validate and reflect upon their methods selected (Sanders, 2016).

The mathematics facilitators content knowledge and also the relations between their knowledge about learners and the knowledge of their learning ways affect learners conceptual learning (Hill and Ball, 2004; Loong, 2014). Hill, Ball, and Schilling (2008) stated that there was an agreement about the fact that the mathematics facilitators who realize effective teaching have the knowledge of learners thinking. Discovering and focusing learners thinking has inherently complex structure. Learners-centered instruction can be implemented if the facilitators reflect their "knowledge of learners thinking" in their teaching.

Anwar, Budayasa, Amin, and Haan (2014) explained that the mathematics facilitators should change their instructions to include the activities that will allow the learners to develop mathematical thinking. Rasiman (2015) stated that a learners will not be able to develop his high level thinking ability well without being challenged to practice in the use of learning. One of high level thinking abilities is critical thinking. The critical thinking can be owned by someone if he is consistently trained through focused discussion or facilitated by an instructor.

Özaltun (2014) examined mathematics facilitators knowledge of learners Thinking, and considered knowledge of learner thinking framework included the nine components and their contents. The components were named as building on learner mathematical ideas, promoting learner thinking mathematics, triggering and considering contradictory thoughts, engaging learners in mathematical learning, evaluating learners' understanding, motivating students learning, considering learners misconceptions and errors, considering learners difficulties and estimating learners possible ideas and approaches. However, Yang and Ricks (2012) highlighted the use of the 'Three-Point framework' by Chinese facilitators in examining crucial events. According to Yang and Ricks (2012), the key point refers to key mathematical ideas of the lesson; the difficult point refers to cognitive obstacles encountered by learners when they attempt to learn the key point; and the critical point refers to the approach that facilitators take to help learners overcome the difficult point. These three points can provide a useful frame for facilitators to focus on the mathematical content, learners thinking, and generate teaching approaches that are associated with both content and learners thinking

Tasdan, Erduran and Celik (2015), stated that this knowledge includes an understanding of what makes learning a specific topic easy or difficult, and the conceptions and preconceptions that learners of different ages and backgrounds bring with them to those most frequently taught topics and lessons. They further, defined one domain of teachers knowledge as knowledge of content and learners. They stated that this knowledge combines knowing about learners and knowing about mathematics. There is need for learner Mathematical thinking to provide an opportunity at that moment for the class to build on that thinking toward a mathematically significant point, (Leatham, Peterson, Stockero and Van Zoest 2014).

Research Objective

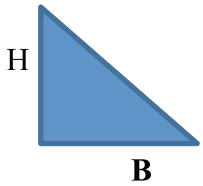
The purpose of this study was to examine the Teacher Knowledge on Students Thinking towards Learning Mathematical Concepts of Area and Perimeter in primary school. Specifically, this research focused on Teacher Knowledge on Students Thinking towards Learning Mathematics.

Results of the findings

Data was collected by observing teachers teaching area of a triangle. The lessons took a span of 35 minutes each. The first session focused on identifying a triangle shape from other shapes. Demonstration was done using a square Manila paper cutouts, the whole idea was to come up with a right angle triangle. The teacher divided the square cuttings diagonally into two, to form a right angle shape. Then, the teacher cut the diagonal line to come up with two right angled triangles. Learners were also given an opportunity, to cut the squares into two and come up with a right angle triangles.

In the second session the teacher introduced, the formula of getting area of a triangle, which was wrongly stated, this shows that the teacher lacked inadequate knowledge on how to introduce the formula of a triangle to the learners for the first time.
Moreover when the teacher introduced the formula of a triangle the teacher interchanged the

Formula instead of saying area of a



is ½ Base *Height

the teachers said the area of a triangle is a ½ height * base. In this case learners will not be in a position to master the correct formula of a triangle and perform more complex task on finding area of a triangle. Here the teacher did not allow learners to construct their own mathematics.Teachers should be encouraged to use formulas correctly to be able to boost students thinking, which will enable learners to reconstruct mathematics and perform mathematics task on area of triangle without difficulty and also to achieve their learning objectives. Use of resources was well used to aid the teacher in defining the shapesThe teacher asked questions to improve learners understanding. The teacher also allowed students to make predicaments.

Conclusion
The government should in service teachers especially on use student thinking, while teaching mathematics concepts to learners, because teachers lack adequate pedagogical knowledge in teaching mathematical concepts. School should encourage peer teaching in primary schools, this will enable learners to improve their knowledge in students thinking. Tasdan, Erduran and Celik (2015), stated that pedagogical knowledge includes an understanding of what makes learning a specific topic easy or difficult, and the conceptions and preconceptions that learners of different ages and backgrounds bring with them to those most frequently taught topics and lessons. They defined one domain of facilitator's knowledge as knowledge of content and learners. They stated that this knowledge combines knowing about learners and knowing about mathematics.

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Problematizing knowledge for teaching

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Assumptions about teaching and learning of mathematics play a vital role in any attempts in conceptualizing mathematics teacher knowledge. In this paper, it is argued that the grounds of research on teacher knowledge show a strong bias toward considering subject matter as an object of teaching (rather than an object of learning). The objectives of this paper are to better understand this shortcoming and to provide potential avenues for resolving it. Two suggestions are specified: (1) aligning teacher knowledge facets toward the promotion of students' learning progression, and (2) taking a model of cognition and learning as the linchpin in conceptualizing teacher knowledge to create a unity among the various knowledge facets.

INTRODUCTION

Despite the relatively short time that research on teacher knowledge has existed as a field, there is quite a multiplicity of frameworks on mathematics teacher knowledge. The last three decades have resulted in demonstrable progress in articulating and capturing what mathematics teacher knowledge is and should be about. Several scholars have significantly contributed to the field mainly in refining, extending, and adding various domains of teacher knowledge, and making them specific to the teaching and learning of mathematics. Indeed, these contributions are crucial pieces of the bigger picture of mathematics teacher knowledge; they have served many purposes quite well and provided empirical evidence that these pieces matter for the specific purposes for which they have been developed. However, the growing acknowledgment of the significance of the theoretical underpinnings underlying the frontier of what seems to have become the adopted perspective in research on teacher knowledge calls attention to the more severe boundaries of our historical ways of thinking (see Scheiner, Montes, Godino, Carrillo, & Pino-Fan, 2017). One set of concerns relates to the instance that, in adopting Shulman's approach of transforming the subject matter, the most-widely used conceptualizations of teacher knowledge take a subject-matter oriented perspective. Though the field provides a multiplicity of frameworks on mathematics teacher knowledge, it seems that we got trapped in particular frames and reference points, and, as a consequence, do not entertain a broad enough scope of differences in opinion and ways of thinking.

In this paper, it is argued for taking a critical appreciation of the origins and evolution of the field since conceptualizations developed in the past may do not support recent advances in research on knowing and learning mathematics. Further, it is argued for reorganizing (rather than accumulating) the frameworks on mathematics teacher knowledge in light of these recent advances. Thus, the intent of the present paper is (a) to raise awareness of habits often taken for granted and left implicit in theorizing and conceptualizing mathematics teacher knowledge, and (b) to revisit our habits in ways that allow to reorganize and redirect the frameworks on teacher knowledge to be more consistent with recent advances in research on knowing and learning.

OLD HABITS: CONSIDERING SUBJECT MATTER AS AN OBJECT OF TEACHING

Many in the field of teacher knowledge today take Shulman's conceptualization of pedagogical content knowledge (PCK) for granted – accepting the view of PCK as an adaption of subject matter knowledge for the teaching enterprise, a process Shulman (1987) called *transformation*. Shulman (1987) stated that

> "the key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students." (p. 15)

The transformation seems to concentrate on the structure and representation of the disciplinary subject matter – in a word, the transformation takes place on the logic of the discipline. The primary purpose of transformation is to organize, structure, and represent the subject matter of the (academic) discipline into a form "that is appropriate for students and specific to the task of teaching" (Grossman, Wilson, & Shulman, 1989, p. 32). Shulman's idea of transforming the subject matter shares certain characteristics with the French and German schools of thought in didactics of mathematics, particularly referring to the theory of transposition didactique (didactical transposition) and the tradition of Stoffdidaktik (subject matter didactics). For instance, Stoffdidaktik is described as an approach that "concentrates on the mathematical contents of the subject matter to be taught, attempting to be as close as possible to disciplinary mathematics. A major aim is to make mathematics accessible to the learner" (Strässer, 2014, p. 567). It is this common concentration on the subject matter of the discipline and the shared idea of 'making the content accessible' that have shaped past and recent approaches in theorizing and conceptualizing knowledge for/in teaching mathematics in a sustainable way.

The philosophy of transforming the subject matter in ways accessible to students underlying PCK has fostered the (almost exclusive) association of subject matter as an object of teaching. From this perspective, subject matter is considered as a sort of package, where the quality of 'making the content accessible' depends on the quality of the vehicles of 'unpacking mathematics content'. The literature identifies various such discipline-specific practices, including, but not limited to, elementarizing, exemplification, and simplification, that require the capacity to deconstruct one's knowledge into a less polished final form where critical components are accessible and visible (Ball & Bass, 2000).

NEW HABITS: CONSIDERING SUBJECT MATTER AS AN OBJECT OF LEARNING

The teaching centered and subject-matter oriented focus in research on (mathematics) teacher knowledge is an artifact of our traditional ways of thinking. However, the primacy of taking the logic of the discipline as the determinant of the transformation process has contributed to a somewhat limited understanding of the complexity of the teaching-learning process. Recent advances in research on cognition and learning of mathematics force us as researchers in the field of teacher knowledge to get out of the frame of our past ways of thinking in conceptualizing and measuring knowledge for/in teaching mathematics. Rather than concentrating on deconstructing and restructuring the subject matter in ways that are accessible to students, a process determined by the logic of the discipline, the perspective is shifted to creating activities and organizing learning environments that are driven by the logic of the students. The teachers' primarily attention is shifted toward how students' knowing and learning progress. To pronounce a shift in emphasis in educational practice from the logic of the discipline to the logic of the students is to proclaim subject matter as an object of learning (rather than an object of teaching). Moreover, it declares to take students' thinking as the driving force in educational decision-making (Carpenter, Fennema, & Franke, 1996). One of the instructional consequences of viewing subject matter as an object of learning (rather than an object of teaching) is not to understand students' conceptions only in terms of misconceptions to be exposed and confronted but to understand students as having productive resources they naturally invoke in some (but maybe not in all) contexts (see Smith, diSessa, & Roschelle, 1993).

Recent work on learning trajectories, local instruction theories, among others converge in the supposition that teachers need to have in mind theories or models about how students think, how their thinking will develop, what problems students are likely to face, and what kinds of responses from the teacher are likely to help them progress. This, in turn, has recently led to a call for a learning-trajectory based instruction (Sztajn, Confrey, Wilson, & Edgington, 2012). Now it becomes apparent that a more unified program of research is needed if we are to acquire an understanding of teaching and learning that will inform the conceptualization of mathematics teacher knowledge consistent with recent advances in research on cognition and learning mathematics.

A MODEL OF COGNITION AND LEARNING AS A LINCHPIN FOR CONCEPTUALIZING TEACHER KNOWLEDGE

Whereas it was essential to initially describe, identify, and attribute various pieces of teacher knowledge and to make progress in obtaining empirical evidence to support each piece of the puzzle, interpreting them in light of a model of cognition and learning allows for the integration of the various pieces into one framework for teacher knowledge. The time has come to move from the accumulation to the reorganization of knowledge facets that are considered as essential for teaching and learning mathematics. Since the alignment of various facets of teacher knowledge toward students' learning progression can be challenging to achieve, a model of cognition and learning of specific mathematical domains is needed that brings cohesion among the various knowledge facets.

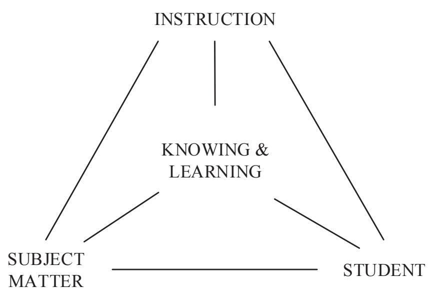

Figure 1: A model of cognition and learning as the linchpin

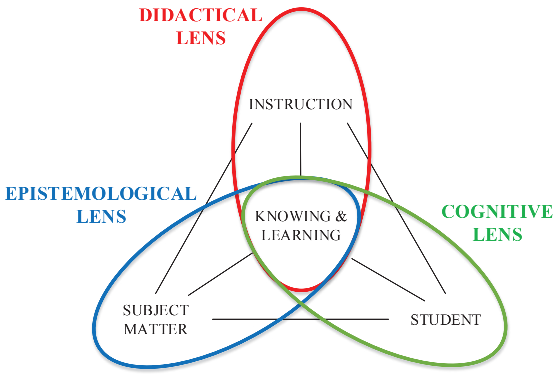

Figure 2: Various lenses on teacher knowledge

In other words, alignment among different knowledge facets could be better achieved if the knowledge facets were derived from a theory-driven and research-based framework on knowing and learning particular mathematical domains or concepts. From this perspective, a model of cognition and learning may serve as a cornerstone that brings cohesion to subject matter, students, and instruction (see Figure 1). The author believes that, in placing a model of cognition and learning at the center of the didactical triangle, the extended didactical triangle, as a whole, provides a useful analytic tool in reorganizing, redirecting, and reinterpreting the frameworks on mathematics teacher knowledge. Various lenses could be identified, including, but not limited to, an epistemological lens, a cognitive lens, and a didactical lens (in addition to a content lens) (see Figure 2).

FUTURE DIRECTIONS IN RESEARCH ON KNOWLEDGE FOR/IN TEACHING

The retrospection on the grounds from where the field has traveled in research on teacher knowledge reveals a strong bias toward considering subject matter as an object of teaching. However, this paper makes a case for resolving this shortcoming. The position paper specifies two considerations that suggest a strong need to move from where we are now to a more advanced plane of thought consistent with recent advances in knowing and learning mathematics: First, accentuating the need of alignment of teacher knowledge facets toward the promotion of students' learning progression offers a critical redirection of existing views and attempts in conceptualizing teacher knowledge. Second, taking a model of cognition and learning as a linchpin in conceptualizing teacher knowledge that creates a unity among the various knowledge facets offers an entirely new point of view regarding this issue. These considerations are unaddressed opportunities for our field to bring coherence and alignment in our increasingly diverse and fragmented research field.

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Resource system of UK mathematics teachers appropriating mastery approaches

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I report on ongoing Ph.D. research exploring mathematics teachers' appropriation of digital resources and the impact on classroom practices in selected UK schools. This qualitative case study examines seven mathematics teachers appropriating resources for 'mastery teaching' in their classroom practice. The study has potential to contribute to the discourse on the challenges of appropriating resources for teaching and learning mathematics.

This study combines an activity theoretic approach (Engeström, 1987) with the more recent 'documentational approach' (Gueudet and Trouche, 2009) from French didactics as theoretical tools for developing an understanding of the teachers' appropriation of digital resources and building up a coherent explanation for its impacts on classroom practices.

It is my hope that the findings from this study will offer opportunity for networking and collaboration on the African continent and share transferable skillsets.

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Michael Achile Umameh

Problem based learning approach for effective preparation of 21st Century Mathematics and Science Teachers in Rwanda

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The introduction of the competence-based curriculum(CBC) in Rwanda by Rwanda Education Board(REB) in 2015 calls change in teacher training for equipping student-teachers with knowledge, skills, values and attitudes that will enable them to bring change in class teaching. In this regard, problem-based learning (PBL) approach as one of the reputed approach to improve the quality of mathematics and science has been introduced in specific subject teaching methods of mathematics and science in University of Rwanda-College of Education(UR-CE). The objectives of this paper are at first PBL approach application in Rwanda teacher training and then examine how the approach was embraced by student-teachers and how it contributes to 21st century skills that are needed for a knowledge-based global society. Data has been collected from 548 student-teachers who were doing subject (mathematics and science) teaching methods in which the PBL was introduced. They have been gathered through observation, survey, and semi structured interview. The preliminary findings of the study reveal a number of skills that are mostly developed by problem based learning approach. Those skills are critical thinking and problem solving, communication, collaboration, creativity and innovation. It has also revealed one of the most challenge for student –teachers which is to set a challenging scenario necessary for PBL approach. Based on the results and the intentions of CBC, the PBL approach should be spread in other teacher training colleges.

Key words: 21st century, PBL approach, Mathematics and Science education, Teachers' preparation

Theme: Quality Mathematics Education for All
Sub-theme: Effective initial and continuing mathematics teacher education

Sub-theme: 4

Integrating information and communication technology (ICT) in Mathematical Education

Indigenous mathematical knowledge, traditional arithmetical algorithms and modern technologies

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Following on from previous studies by Paulus Gerdes on the Sona sand designs, in this article we introduce some way of teaching the algorithm of finding the greatest common divisor of two integers. In doing this we show, in particular, how to combine traditional algorithms with modern technologies. A geometrical version of the Euclidean algorithm and an unconventional algorithm to verify whether a positive integer number is prime are also presented. The algorithms are introduced through different dynamic software, that make use of mirror curves, closed lines including points of a given grid.

THE AIM OF THE RESEARCH AND ITS THEORETICAL FRAMEWORK

The educational objective of the teaching proposal described in this article and of its piloting is to create the conditions so that the concept of gcd - greatest common divisor, of strictly arithmetical nature, is introduced from a problematic situation apparently of geometric nature, being connected to the design of lines to be drawn in accordance with certain rules assigned.

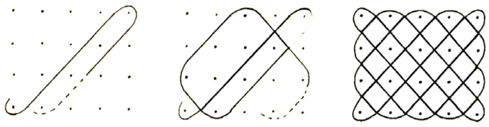
The task of tracing the lines, initially done in manual form by students, is then proposed to be entrusted to a simple software that allows students to calculate the gcd. The introduction of this software in the classroom helps create the conditions for the realization of a germinal micro-world, where technology assists and supports the emergence and development of students' mathematical thinking. In fact, using this software creates, in turn, the conditions for the emergence of the classroom reflections, questions and demands concerning the gcd concept for which the introduction of additional software may represent the teaching instrument that fosters the consolidation of this concept.
The theoretical reference of this paper is basically on two strands:

- studies conducted on mathematics associated with Sona designs, belonging to the cultural heritage of the Angolan people;
- studies related to the introduction of new technologies in the mathematics classroom, with particular reference to the creation of micro-worlds.

The Sona sand drawings

With his studies of Sona drawings, made in the sand by storytellers belonging to the indigenous cultures, Paulus Gerdes (1999, pp. 156-205) shows that some elements of the school mathematics knowledge are inherent and identifiable in the tracking of these drawings.

The most elementary Sona are represented by lines drawn around points arranged in rows and columns, so as to form a grid. The rules for drawing these lines can be derived from the examination of Figures 1.



Figures 1

Gerdes shows how the number of closed lines that is necessary to trace to enclose all the points of a grid (p, q) is the gcd(p,q).

This result has already provided the opportunity (Favilli & Maffei, 2006) to design an educational activity that allows students to get to the same result in an experimental way, while introducing technology in the classroom:

- identification of the rules to draw the Sona from the request to complete a path partially drawn in a grid (5,4) of points;
- drawing, with paper and pencil, of a few (p,q) Sona, with p and q natural numbers greater than 1 and less than 20;
- identifying, for each pair (p,q) used, the number n of closed polygonal lines (mirror curves) that have been necessary to enclose all the points in the grid;
- introduction of Sona_Polygonal_1.1 software (Figure 2) to draw some different (p,q) Sona with p,q < 100 and count the number of the necessary polygonal lines;
- construction of a table showing the different (p,q) used and the resulting n;
- analysis of the table to get to find the relationship n=gcd(p,q);
- introduction of the Sona_GCD_1.0 software (Figures 3) to compute the gcd(p,q).

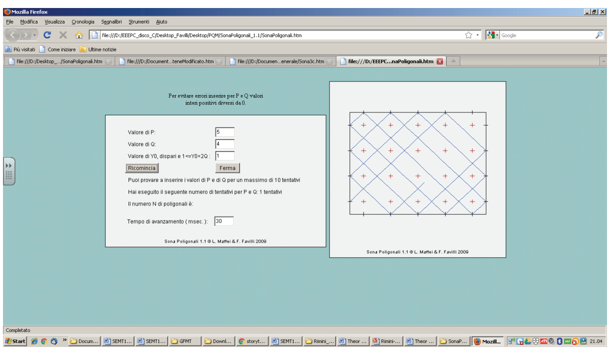


Figure 2

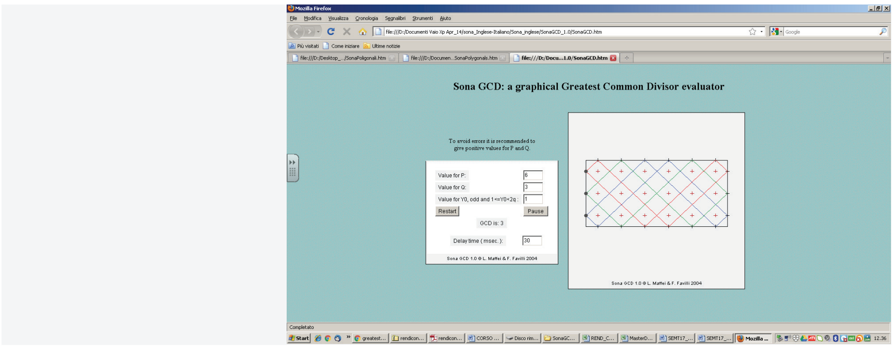


Figure 3

The activities outlined above are carried out first individually and then are discussed and confronted in the classroom, the teacher taking on the role of moderator and facilitator.

Micro-worlds and technologies in mathematics education

For the introduction of the notion of micro-world and its use in mathematics education we refer to Balacheff & Kaput (1996, p. 471):

> A micro-world consists of the following interrelated essential features:
>
> i. - a set of primitive objects, elementary operations on these objects, and rules expressing the ways the operations can be performed and associated - which is the usual structure of a formal system in the mathematical sense.
>
> ii. - a domain of phenomenology that relates objects and actions on the underlying objects to phenomena at the 'surface of the screen'. This domain of phenomenology determines the type of feedback the microworld produces as a consequence of user actions and decisions.

In a micro-world students are stimulated by genuine problem-solving activities, which drive them to formulate and test mathematical ideas without the constraint of explicit formal presentations, to develop and use mathematical ideas in the solution of a problem.

The Didactic Proposal, Its Methodology And Piloting Outcomes

The didactic proposal object of experimentation aims to create a micro-world in the mathematics classroom, where technologies are to support the introduction and development of the semiotic concept of greatest common divisor of two non-zero natural numbers.

The teaching unit requires the availability of a personal computer for each student.
After the introduction of the concept of gcd in the above described way, the teacher shows another algorithm which allows the calculation of the gcd without the need to use the concept of prime number, which definition may then be postponed: the Euclidean algorithm.

This algorithm, introduced by Euclid as the solution to the Proposition VII.2 in the *Elements*, can be described as follows:

- if p<q, exchange p and q.
- divide p by q and get the remainder, r; if r=0, report q as the gcd(p,q).
- replace p by q and replace q by r, and return to pre previous step.

To make better understandable this algorithm and the consistency of its results with those obtainable through the procedure already introduced through the Sona, the teacher can then introduce the Sona_Euclid_1.0 software that provides a visual and geometric representation (Figure 4) of the Euclid's algorithm, identifying the gcd(p,q) as the minimum size of the different squares through which it is possible to fully cover the rectangle of (p, q) size.

This second step of exploration and consolidation of the concept of gcd, terminated not being necessary to precede it by the definition of prime numbers in view of the use of the common algorithm for the calculation of the gcd between two natural numbers p,q > 0, which requires their previous prime factorization.
The chosen teaching path for the introduction of the gcd and the use of two software can then push the teacher to reverse the order in which prime numbers and the gcd are usually introduced: first, the gcd and then prime numbers, instead of first prime numbers and then the gcd.

In fact we can get to say that a natural number p is a prime number if gcd(p,q)=1 for every natural number q<p! Moreover, with simple arithmetic considerations, we can say that p is prime if gcd(p,q)=1 for each natural number that is less than or equal to sqr(p).

With such a definition, equivalent to the traditional one, it is then possible to stimulate the interest of students in search of natural numbers that are prime. Their interest can be further developed by offering them the possibility to use and carefully examine the procedure performed by another software, Sona_Erathostene_1.0, where the reference to Eratosthenes is improperly dictated by the procedure he introduced (the so-called Eratosthenes' sieve) to identify the prime numbers that are less than a given natural number.
The Figure 5 illustrates the procedure followed by the software to verify that 59 is a prime number:

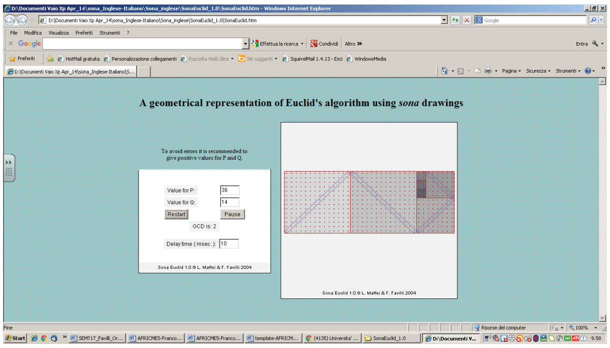


Figure 4

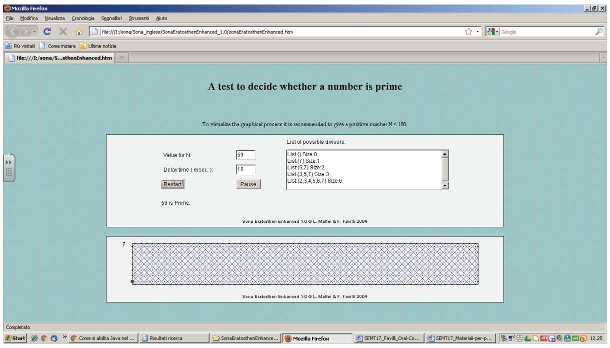


Figure 5

The piloting of the above described teaching unit has proved to foster, in the classroom, the construction of mathematical ideas from particular situations that affect students' argumentation and expression of ideas.

The technological setting showed to promote the development of a collaborative environment in the classroom, where the advancement of the students' strategies and the development of their mathematical knowledge can be better identified and analysed.

Furthermore, the different software (*artifacts*) were for the students effective instruments that allowed them to associate schemes and techniques to the instruments, when undertaking the given tasks. (Rabardel, 2002)
As far as students' affect and attitudes are concerned, a few meaningful comments of theirs can show the positive impact of the piloting:

- We could work relaxed and freely, and it was something fun and unusual.
- I loved all lessons on Sona. I liked everything in these beautiful activities.
- This teaching unit has helped me better understand the mathematical laws (as they are, what are they for).

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Using technology to support mathematics education for learners with vission loss. Lessons from mwangaza project, kenya

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Technology has bridged the learning divide among learners from different backgrounds. Technology has increased access to learning resources hence reducing dependency on teachers' pedagogical proficiency. STEM education has benefited most from technology in supporting learners conceptual understanding However, a quick observation shows that learners with vision loss disabilities are absent from mainstream STEM related courses at tertiary level. This absence provides an indicator that VI learners experience of science and mathematics subjects at school level has not been successfully adapted to their needs. STEM education for blind and low vision students the world over, and similarly in Kenya, has been held hostage to a combination of fear, doubt, lack of knowledge, lack of teacher training skills and ***resources****. This article shares findings from a study that used accessible software with the visually impaired learners with the aim of initiating informed discussion and possible solution process towards inclusive Mathematics Education for learners with vision lose and other related disabilities*

Introduction

Daily encounters include interpreting our surroundings through numbers and values usually presented in charts and graphs. These include weather forecasts, stock market reports, public opinion polls and economic indicators. These daily encounters are comprehensible for those who have had the advantage of STEM education. STEM Education opens doors of confidence and explorative possibilities for ALL learners. This article shares findings from a study that responded to the research question, " To what extend can technology afford visually impaired students opportunity to access STEM knowledge?".

A collaborative research study on the impact of selected accessible software for STEM learning by visually impaired students was mounted for all students with vision lose in Kenya. Two accessible software programs, the Accessible Weather app and the Accessible Website /portal were targeted for this initiative. The aim of the initiative was to determine the extend to which VI students engaged with this software, and teachers ability to sufficiently use these experiences to unpack STEM related concepts across the grades. The study was motivated by the observation that,of insignificant number of VI students who access university education, non of them opts for STEM related courses. This pointed towards possible missed opportunities of studying STEM subjects in their formative years in school

This study was a shared effort between: the Sonification Lab at the Georgia Institute of Technology ("Georgia Tech") in Atlanta, USA and Kenyatta University, in

Nairobi, Kenya. This was a two-phase study including (1) a nation-wide survey of the interests, needs, skills, and opinions of blind students and their teachers, with respect to information and communications technology (ICT); and (2) initial development, deployment, and evaluation of some novel assistive technologies that represent potential new approaches to STEM education for students with vision loss. The survey also included university students with visual impairment

The study thesis was based on how to make data and thereby Science, Technology, Engineering, and Mathematics (STEM) more accessible to blind students and teachers. The main focus in this line of research was the study of auditory graphs and the development of software tools to support the use of multimodal data displays in the classroom. Several novel software tools and educational approaches that hold great promise for STEM education amongst learners with visual impairment such as MathGENIE and the Sonification Sandbox have been shown to enable Math and STEM education and improve efficiency for the teachers.

While there have been projects in the past aimed at helping educate blind students in Kenya (and elsewhere), and while some projects have attempted to make computers more available, this specific study used a blend of education research, technology, training, and accessibility, rolled together with the deployment of both computer labs and training and research with a transformative object on an international scale

The purpose of this study was to determine if perhaps these benefits could be realized by a much larger group of learners, if the tools and techniques were more broadly deployed. The tool developed for intervention from the Sonification Lab's used modern computers, assistive technology including electronic devices as well as Braillers and other resources. While the potential for widespread benefits is clear, it is crucial to be able to leverage a population of learners and teachers with knowledge, desire, and access to technology. This was a particularly challenging constraint in a country such as Kenya, where technology and ICT skills can be scarce.

The ultimate vision, still remains that once students and teachers are trained to use the technology, STEM education tools in general and mathematics tools in particular can be deployed to expand the education of those students. The combination of marketable computing skills and a better, more complete education (including STEM topics) can improve the careers and lives of blind individuals across Kenya.

However, for the tools and methods deployed to be developed with the local context in mind this study identified the specific needs of the Kenyan VI students at tertiary institution using Kenyatta university students to adjust (or develop) tools and resources that were appropriate for blind and low-vision Kenyan learners..

First, it was important to understand the experience of blind students in Kenya with respect to technology, their perspectives on career choices, aspirations, and other psycho-social measures. As a fact most of the students in Kenya have very little technology experience and very few resources, this state of technology experience was critical as a baseline against which the success of the intervention was measured , there were simply no such data available hence the need to conduct a nation-wide baseline survey of the technological skills and experience of the blind and low vision students in Kenya and university students.

Then, the second phase of the project deployed the GT Sonification Lab STEM education software, curriculum, to students along with appropriate training and practice using tablets configured with the accessible software. Besides anticipated gains in computer skills and STEM knowledge, improvements in career aspirations and students' perspectives on their role in society, new projects that could conduct additional software development and deployment for a range of tools related to weather, educational games, and mathematics education, among others were envisioned.

Finally, a training program to teach employable skills (e.g., software programming; Web Accessibility Assessment) was developed, showcasing the fact that (blind) students are able to seek technology-supported employment.

Background in Assistive Technology and STEM Education

Every day we need to understand data in order to make choices in our lives. For people with vision loss, the typical graphical presentations of data may be difficult or impossible to access. As a result, education and employment are difficult for blind individuals, especially in STEM fields.

The development and deployment of technology, training, and STEM education tools for the blind remains the one stop solution for inclusive learning in sTEM subjects in Kenya, especially among school-aged children as well as university students. Unfortunately, technology and STEM education for the visually impaired in Kenya has been constrained by a lack of resources and experience. Even though Kenya's school system includes at least twelve (12) schools for the blind, each with hundreds of students; plus thousands of additional low-vision students who attend "integrated" public schools, there are a negible % (0.002) students who access tertiary education and all of whom take non sTEM related courses inspite of their capabilities.. *While there have been projects in the past aimed at helping educate blind students in Kenya (and elsewhere), and some projects that have attempted to make computers more available, none that we are aware of has the blend of education research, technology, training, and accessibility, rolled together with the deployment of both computer labs, tablets and training, scale.*

This article is delimited to those learners in the education system (school and university) students only. It does not include learners who have completed their education or are out of school

The study was replicating a the locally contextualized GT-developed auditory graphing software, along with bone-conduction audio headsets,which has changed the way math teachers at the Georgia Academy for the Blind (GAB) interact with their students, allowing the teachers to spend less time lecturing, and allowing the students to spend more time interacting with each other and with the teacher, during more hands-on practice, exploration, and learning.

The survey

Before the intervention, a nationwide survey of blind and low-vision students at schools across Kenya, was carried out. The purpose of the survey was to begin to collect data about learners in Kenya with vision loss. Data was collected on demographic information, computer and technology experience, interest in computer training, and various measures of life satisfaction, psychosocial status, and career aspirations. This data serves as an effective baseline against which to assess the efficacy of computer training programs, and projects to deploy assistive technology as part of classroom education for this population. Data was also collected from older undergraduate students with vision loss, at Kenyatta University, and from teachers who work with blind and low-vision students. This is the data from which the following findings are shared.

Data Collection Method

The method of research used during the data collection was both interview and questionnaire oriented. Computer Assisted Personal Interviewing (CAPI) which is a computer assisted data collection method was used in survey data collection, using a portable personal computer such as a tablet. Effective use of CAPI often results in quick turnaround surveys,

Students with vision impairment voluntarily took part after a careful process of informed consent.

The students were explained the reason for the study and its potential benefits to the visually impaired and society as a whole. The purposes of the study included: an assessment of the students' experience with, and knowledge of technology; their career aspirations and interests in learning technology; and to begin to probe their attitudes towards their role in society (and whether technology might help improve things). The following tables provide some descriptive frequencies of how many students were in each grouping for all of the comparisons we are interested in.

Data were collected, geo-tagged, encrypted using industry-standard protocols, and sent directly into a database on a secure server, making the process quite reliable and efficient. The data collection hardware, software, and servers were provided by Infotrak Research & Consulting (http://www.infotrakresearch.com), a Kenyan survey and data collection company.

Results from Nationwide Survey of Students With Vision Loss

All data was analyzed using IBM's Statistical Package for Social Sciences (SPSS) software. Responses from all students who participated were anonymized and compiled into a master response list. There were a total of 23 valid cases used for analysis. To look for trends in students' responses a series of composite scores were created. These scores were created out of subset questions on the questionnaire. Individual scores from each item within the composite groups were added together for the composite score. We took this approach to make it easier for group comparisons and for looking for overall trends.

When VI undergraduate students were asked about computer use, 53.1% of students responded that they use a computer every day. All students responded to using a computer at least a few times in the past or more.

Self-Reported Visual Difficulties Scale

Number of valid cases	32
Missing / Omitted cases	0
Mean	18.13
Possible range of scores	6 to 30
Standard Deviation	4.18

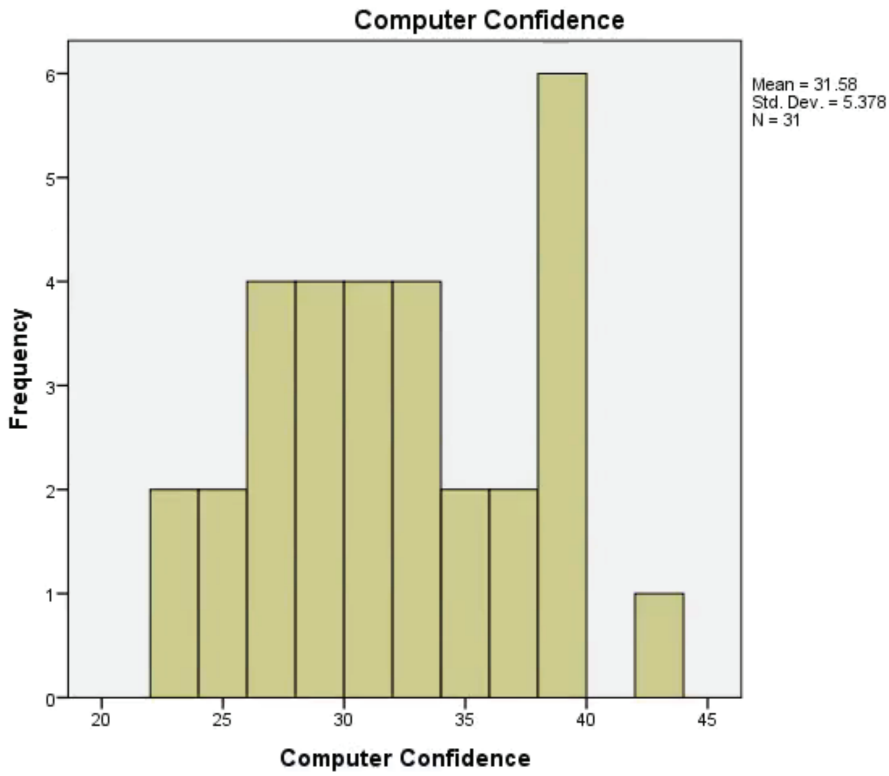
The frequency table of Self-Reported Visual Difficulties Scale for students in the supplemental survey of undergraduate students.

Perceived Burden Scale

Students were asked how they felt about themselves using the item; "I feel ashamed or embarrassed…" and "I often feel that I am a burden on others…". on a 6-point Likert scale from 1 (strongly disagree) to 6 (strongly agree) with the statement. The purpose of these questionnaire items is to establish the level of students' self worth. If these scores started out high we would hope they lower over time, and if they started low we will look to keep them low. This all would indicate that students would tend to see themselves as less of a burden to others as they become more proficient at using computers. The composite score for *Perceived Burden* is on a scale from 2 to 12 revealed a man score of 4.69

Responses for the *Interest in Training* score ranged from 10 at the lowest to 60 being the highest amount of interest wit a mean of 54

According to attitude—behavior theory, it has been hypothesized that computer use enhance beliefs about self-perceived computer confidence, which in turn affect *attitudes towards computers. A study by Leven in 1995 on self-report surveys that measured these* three constructs revealed that (a) computer use positively affected computer confidence, and (b) computer confidence positively affected computer attitudes. Unexpectedly, direct computer use had a negative effect on computer attitudes, when confidence was held constant. Results suggest how computer educational environments are important for confidentce building. This study sought to measure respondents computer confidence using a composite score . The *Computer Confidence* score ranged from 7 at the lowest to 42 as the highest amount of confidence, returning a mean of 31.6. as follows



The Frequency histogram of Computer Confidence Scale for students in the supplemental survey of undergraduate students.

Fluid vs. Fixed Intelligence Scale

The Fluid versus Fixed Intelligence scale was a pair of items that were added onto the questionnaire to measure if students see intelligence as something fixed (you have a set amount of intelligence and cannot change it) or if it is something fluid (you can become more intelligent with hard work). We obviously wanted the students to feel that intelligence is something fluid and that ‘everyone can all learn new things’. This composite score ranged from 2 to 12 points so that “Intelligence is more fluid” = 2; “Intelligence is more fixed” = 12. The mean score was 5.19 with an SD of 2.7

The undergraduate students at Kenyatta University showed similar distributions of responses as the students at the younger ages. The undergraduates generally report slightly better scores on functional vision (less impairment) than the younger students, which is not surprising since the younger sample includes many blind students whereas the students who reach university are much more likely to have some vision (though certainly not all do). Further, the older students continue the trends seen in the primary versus secondary schools, in that the undergraduates show lower perceived burden. They also report more computer usage, higher perceived skills (which is likely accurate, given their more frequent usage), and overall greater interest in technology skills training.

In addition, teachers observed that the reality of integrating technology in the classroom rides on uninformed assumptions. According to them, the present status is that teachers do not have access to computers even when they are in the school lab. Teachers have no skills and have no support on the ground to engage with assistive technology at the level provided by policy expectations. During the discussion it became clear that many teachers do not own smart phones which could be an alternative source to accessing technology. One of the teachers had this to say….*". Who said that a blind person can only learn through audio channel? Let us think broadly and come up with technological innovations which can effectively assist blind persons."*

Teachers also noted that technology makes the lesson exciting and learners are able to concentrate and hence learn better. That technology affords learners autonomy to learn by themselves especially through the Internet and also share ideas with other learners elsewhere about a particular content through technology.

Challenges anticipated with the integration of proposed accessible software

Prior to intervention, teachers were asked to discuss about challenges they anticipated with the intervention of integrating the accessible software programs in the instruction for learners with special educational needs. Several constraints were listed among those that were common were that, schools do not have computers enough for all teachers and children and those that were available were not accessible for teaching purposes. The other constraint was related to the time for training with the present heavy teaching workload. The added cost of hiring technicians to manage and support the process, the frequent power blackouts and managing learners with multiple handicaps were all raised as possible integration challenges. Other challenges include lack of room for storage of devises and for training. Blind students, it was reported, "*of course don't do chemistry, physics and even pure biology"*. This is the issue as was noted by teachers was that science related subjects are full of visuals. Most of the STEM content is presented in a concretized format and requires a learner to relate what she/he is learning to what the learners has experienced before. The VI lived experiences of the real world is minimal or relatively abstract. It is then very difficult for the blind people to understand most of the content in mathematics and sciences unless STEM curriculum content need to adjusted for VI learners

It was observed that even teachers who are recent graduates had no understanding of how a blind person perceives things. They teach as if they are teaching sighted. The children feel completely discouraged. Also discussed was the lack of resources adapted for the blind to help the learners master the contents in mathematics and sciences.

The teachers felt encouraged that enhanced computer based technology by considering accessible software would enable VI students compete with sighted learners in STEM because using Braille was slow to read and made covering of mathematics/science syllabus difficult. It was accepted as obvious that mathematics and sciences subjects are difficult for both learners but is worse for the blind learner. The teachers indicated that besides the many visuals used, the language is very abstract. Words like distance, height, length, width etc which a sighted person because he can see is easy to understand made no sense to a totally blind person.

According to one blind teacher, the struggle to teach science and even mathematics to the children who are blind is double; he states that, "*Like me, I am a blind teacher and the way I was taught science I didn't grasp some concepts properly. Then I am supposed to teach the same to the children; this is just pretence. How do we expect the children to learn?*" According to him, only technology will lift the frustration he feels.

During the discussion, it was said that, a blind person might not value some of the knowledge in mathematics and science such as trigonometry in math, triangles, and circles. The children believe that knowledge is not applicable in their life. Teachers stated that blind learners and especially children would like to have knowledge of what can solve their immediate problems. To them some topics in mathematics and sciences are not related to their immediate needs and hence they resist learning such topics. However when teaching them on how to count money or about body grooming ,they get excited and eager to learn. Weather concepts may also be of interest, given their applicability.

At secondary school level, teachers observed that the learners perform very poorly. Knowledge gaps exist in mathematics and science-related subjects and these students have a negative attitude towards STEM subjects.

Teachers requested that to help the blind children to learn sciences "*we must make sure that teachers of sciences are well versed in teaching the sciences to the blind students and also make sure that the technology for the blind is availed to both the teachers and the learners".*

They believe that mathematics and sciences requires a lot of sight, and that the nature of these subjects requires direct mastery of the environment. Teachers added that if was possible to come up with technology that can help the blind to master this then a lot of justice would be done to these learners.

One other factor that came through was that the blind students in Kenya lack role models, especially with respect to science, technology, and technical fields. Majority of the blind do not or in essence are not able to pursue mathematics and sciences at higher levels of education. The students therefore lack someone to emulate.

The propagation of the notion that Blind students cannot do STEM related subjects or careers is a misnomer. The truth is that teachers are yet aware of how to support these cadre of learners VI teachers are familiar with talk- back (assistive) Technology such as JAWS but have not used it much. Although teachers appreciate that such technologies will boost learning-teaching of the STEM subjects, it is the weather App that holds the promise.

.During a Focus group session, caution was raised on how technology gets deployed,

Deployment and Evaluation of Accessible STEM Software Tools

One of the major long term goals of the Mwangaza Project is to leverage the computer skills that students and teachers will eventually be gaining, to enhance the teaching of core curriculum in Kenyan schools. That goal will clearly require participation of the Kenyan government, since delivery of curriculum falls under the purview of the Kenya authorities. As efforts are ongoing in that regard, a parallel effort is beginning in the Mwangaza Project team. Specifically, accessible weatherAppwe is a novel software that can be used in creative ways to support teaching, particularly STEM content. New software applications include stand-alone software, Web resources, and apps for mobile devices. It should be noted that there are a great many existing software tools available to teach STEM, and many of them are accessible to one degree or another. sourcing and using those extant tools is a different initiative, but creating resources from scratch, using iterative and participatory design approaches was the purpose for this intervention.

The development process had such steps as having target users interact with various versions of the software, complete benchmark tasks, and provide feedback about the accessibility, usability, and ultimate utility of the software. This kind of interaction was started by deploying and evaluating one of our software tools, the *Accessible Weather App*, available in the Android store.

The Accessible Weather App

For many, checking the weather forecast is part of the daily routine. Staying informed about current and upcoming weather conditions is especially important for users with visual impairment because the decisions they make about their route, wardrobe, and assistive technology choices for the day can have grave impacts on their daily commute. For instance, knowing that there is a high probability of rain may allow the user to preemptively select a different white cane, to make it easier for navigating with large puddles on the ground, or it may help remind them to bring a raincoat. Unfortunately, the most popular and reliable weather apps have not been designed with accessibility for visually impaired users as the top priority. For a fuller discussion of the need for, and the design of, the accessible Weather App, see Tomlinson, et al. (2016).

While the screen reader accessibility features on mobile devices may allow users with visual impairment to glean information from their device, this does little to provide an *equal* user experience since most of the weather apps include large amounts of visual information, or buttons that lack text descriptions (Rodrigues et al., 2015). Additionally, a user reliant on the screen reader is forced to consume the information in a preset order. Often this results in additional time or steps wasted to get to the intended information, such as current temperature, overall conditions, or the upcoming forecast. While this lack of efficiency and flexibility often results in a poorer user experience, in some cases weather information is completely absent for the user. If the current condition is presented as a weather icon, the screen reader is likely to just skip over it all together.

Thus the GT Sonification Lab set out to design a weather app from the ground up, with accessibility for users dependent on screen readers as the top priority. While the approaches and design decisions for both the screen reader friendly information layout and the auditory weather icons that was implemented in this app can extend to other types of information and apps, focus was on a weather app because of it could provide the greatest amount of impact through an application that would be used on a regular basis. Background research that preceded the actual development included assessing the current status of the weather apps already available, surveying users to find out their needs and wants, designing and building an app for beta testing, and then iteratively testing and re-designing portions of the app. Finally, once the app was completed, it was deployed in Kenya for some evaluation.

Evaluation: Deploy and Interact

A demo using an android tablet with external speakers to let the students have a feeling of what is expected of them in terms of interacting with the app was done. After the demo, the app was downloaded from their phones and the questionnaires were completed by the students once they had interacted with the app.

After analysis by the Georgia Tech Team, a follow up session to determine frequency and comfortability of using the app was made. The results from the analysis is discussed below. To evaluate the Website, 8 Ku students were taken through the accessibility use and transfer of knowledge by the KU team . Interviews on the three aspects were carried out and is herein attached. The group was strategically selected to include student with low vision and those who were blind.

Generally students felt more comfortable with the app than the website

Evaluation: Survey

After data from all the surveys were compiled for analysis. It should be noted that these results must be treated with some caution, since there was a broad range in the level of student, their vision level, and most importantly the extent to which they were able to interact with the app. In general, some of the students had a more structured and more extensive interaction with the device and the app.

Results

A total of 85% agreed and strongly agreed to use the system frequently. This is a likely indication of the dire need for the system app, and how it strongly appealed to the respondent

From the statistics; 50% found the system App simple to work with as compared to the 40% who found it complicated.70% agreed that the system was easy to use. Generally 80% agreed to the system as compared to the 20% who disagreed. 40% of the respondents did not see the need for the technician as compared to the 10% who needed a technician. This is a likely indication that the system is user friendly.75% of the respondents found the system well integrated as compared to 5% who felt that the system was not well integrated. 55% of the respondents did not find too much inconsistency in the system as compared to 45% who felt that there was too much inconsistency in the system. A likely indicator for fine-tuning. 75% agreed that most people will learn to use the system very quickly; with additional 20% somewhat agreeing. 30% found the system very cumbersome to use as compared with 70% who did not find the system very cumbersome to use. 90% of the respondents agreed and strongly agreed that they felt confident using the system. 55% of the respondents felt that they did not need to learn a lot of things before getting into the system as compared to 45% who needed to learn a lot of things before getting on with the system.

Discussion of App Evaluation

The amount of time spent actually interacting with the app, and the formality of the interaction (including training, benchmark tasks, individual use of the device/app, and completion of the survey) varied considerably.

As seen in the data above, the students generally reported that the app seemed accessible, useful and usable. There were some issues that were identified, and those have already been addressed in subsequent versions of the software.

The plan has been that as one very simple example, a primary school mathematics teacher at a school for the blind could assign one student each day to consult the weather app, and record the minimum and maximum daily temperature for that day. Then, after a week, the students could report their numbers to the teacher, thereby creating a classroom “data set” of weather observations. The teacher could then use that data set to teach foundational mathematics concepts of minimum, maximum, average, mean, median, mode, trends, and so on. They could even learn how to create graphs of the temperature data (using accessible graphing software such as MathGNIE from Georgia Tech). Students participating in the construction of their own learning experience can be more engaged, and may have a closer connection to the practical utility of the STEM content (i.e., mathematics, in this case). This kind of teaching module development, and actual deployment in class in Kenya, is planned for subsequent phases of the Mwangaza Project, beyond the scope of the current report.

Website usability and accessibility evaluation by university (KU) VI students

KU –students had no problem in accessing opening the site and navigating the process. It was easy for the partially blind bout difficult for the total blind. The greatest huddle for the total blind was to enter the address. But when the site is open, the navigation was manageable. In effect, the partial needed minimum assistance while the total blind needed maximum help.

When comparing manipulation of the APP and the site and the majority favour the APP. Which they found more direct and flexible. Suggested modifications on the operations when using the site included magnification of the labels (that come to the screen); the terms used (e.g. vertical lines) especially if to be used at primary level.

On how to use the APP or website for learning of Mathematics the following suggestions were proposed; a talk-back calculator with headphones to avoid 'noise' in the classroom. Some tablets have talkback calculators which can be useful for the purpose.

Summary

Students were able to manage to open the site and navigate easily. This was much more so for the partial Vis. The total blind needed a lot of assistance.The total Vis seem to be comfortable with the APP on the tablets than going to the site.At primary school level, the Apps were preferred.

The site opens quickly if instructions are followed. It would be useful to try out the site experience with the school pupils-both at Primary and Secondary levels.

Overall Discussion and Conclusions

This initiative is an amazing blend of education research, technology, training, and accessibility, rolled together with the deployment of both computer labs and training, and with the support of major research universities, corporations, and the government's education department. This effort is intended to be a truly transformative project, on an international scale. The initiative address individual needs for learners with vision loss while at the same time increasing learning opportunities in STEM education.

Evidence of computer skills training is having important impact not only on skills and computer confidence, but also in terms of the psychosocial well-being of the students who have received training.

The teachers have made it clear how much they also value computing skills, but additionally expressed their opinions regarding training, support, and careful deployment. These are issues that must be carefully considered in any technology deployment effort,

Finally, providing computing resources (labs), and training students and teachers in their use, is only the first (albeit crucial) part of any technology evolution. The real value comes when those technology resources can be used effectively to enhance the general education of students in Kenya, especially in the STEM subjects that have traditionally proved most challenging for blind and low vision students. Deploying software tools that already exist, and developing (and evaluating) new software tools to supplement, is the next step in leveraging technology. We look forward to continuing the process of deploying such tools, and working closely with teachers (and education officials in the Kenyan government) to develop teaching modules and strategies to make effective use of the tools in their classes.

This results reported in this project are just the beginning. There remains so much more to do, but we move forward from the solid foundations of research and evidence-based design laid thus far.

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The use of visual mediators in the learning of subgroups

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The proposed study, which is a ramification of a larger study on the learning of Group Theory, aims to shed light to the ways undergraduate mathematics students use visual images in order to enhance their mathematical learning. In particular, this study aims to examine how these students use visual images in order to endorse the notion of subgroup. For the purposes of this study, the author uses the Commognitive Theoretical Framework, which considers visual images (or mediators) as one of the four essential characteristics of mathematical discourse. The collected data includes students' coursework and examination scripts, and student and staff interviews. Results suggest that at these initial steps of students' encounter with Group Theory, the use of visual mediators embraces certain difficulties that need to be overcome. These difficulties do not necessarily stem from Group Theory per se, but usually are due to incomplete metaphors from other mathematical discourses, such as Complex Analysis.

BACKGROUND

Undergraduate mathematics students' first encounter with Group Theory is more often than not a laborious assignment, partly due to the abstract nature of Group Theory (Hazzan, 2001). A typical first Group Theory course requires a deep understanding of many abstract mathematical notions. The learning of the newly introduced notion of group is often an arduous task for novice students and causes serious difficulties in the transition from the informal secondary education mathematics to the formalism of undergraduate mathematics (Nardi, 2000). Consequently, the introduction of the notion of subgroup, and the Subgroup Test[7], is a significant milestone in an introductory course in Group Theory, and it is necessary to investigate further (see Ioannou, 2018). This study aims to explore the ways undergraduate mathematics students use visual images. For this purpose, the author uses the Commognitive Theoretical Framework (CTF) (Sfard, 2008).

[7] Subgroup Test refers to the affair of proving that a given algebraic structure is non-empty, closed under operation, and closed under inverses.

METHODOLOGICAL AND THEORETICAL FRAMEWORK

This study is part of a larger research project, which conducted a close examination of Year 2 undergraduate mathematics students' conceptual difficulties and the emerging learning and communicational aspects in their first encounter with Abstract Algebra (see Ioannou, 2012). The course was taught in a research-intensive mathematics department in the UK. It was mandatory for Year 2 undergraduate mathematics students, and a total of 78 students attended it. It was spread over 10 weeks, with 20 one-hour lectures and three cycles of seminars in weeks 3, 6 and 10 of the semester. The role of the seminars was mainly to support the students with their coursework. The course assessment was predominantly exam-based (80%). In addition, the students had to hand in a threefold piece of coursework (20%) by the end of semester. The gathered data includes the following: Lecture observation field notes, lecture notes, audio-recordings of the 20 lectures, audio-recordings of the 21 seminars, 39 student interviews (13 volunteers who gave 3 interviews each), 15 staff interviews (5 members of staff who gave 3 interviews each), student coursework, and student examination scripts. For the purposes of this study, the data of the 13 volunteers has been analysed, following the principles of Grounded Theory (Glaser and Strauss, 1967).

For the purposes of data analysis, the author has used the CTF. According to CTF, mathematics is considered an **autopoietic system of discourse**, namely "a system that contains the objects of talk along with the talk itself and that grows incessantly 'from inside' when new objects are added one after another" (Sfard, 2008, p129). CTF defines discursive characteristics of mathematics as the **word use**, **visual mediators**, **narratives**, and **routines** with their associated metarules[8], namely the **how** and the **when** of the routine. Sfard (2008) describes two distinct categories of learning, namely the **object-level learning** (expansion of the existing discourse attained through extending a vocabulary, constructing new routines, and producing new endorsed narratives) and the **metalevel learning** (involves changes in metarules of the discourse). Moreover, learning involves a discursive shift that often causes **commognitive conflicts**, which are defined as situations that arise "when communication occurs across incommensurable[9] discourses" (Sfard 2008, p. 296). Commognitive conflicts should be considered "a gate to the new discourse rather than

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[8] *Metalevel rules* "define patterns in the activity of the discursants trying to produce and substantiate object-level narratives" (Sfard 2008, p. 202)

[9] Incommensurable discourses are the discourses that differ in their use of words, visual mediators, routines or their rules of substantiation. In addition, they may allow the endorsement of seemingly contradictory narratives, due to the fact that they do not share criteria for deciding whether a given narrative should be endorsed or not. (Sfard, 2008)

a barrier to communication", where "both the newcomer and the oldtimers must be genuinely committed to overcoming the hurdle" (Sfard 2008, p. 282).

DATA ANALYSIS

Data analysis suggests that, contrary to what was expected by the teaching team, the use of visual images, offered to enhance students' mathematical learning, did not have the expected impact. This study investigates instances of problematic use (or absence of use) of visual images in students' solution of exercise 3 of the first part of the coursework, abbreviated as CS1E3 (see Figure 1). Data analysis identifies problematic use of visual images in the solutions of eight of the thirteen students.

3. Using the usual test for being a subgroup, give proofs of the following:
(i) For any $n \in \mathbb{N}$, the set $n\mathbb{Z}$ of integers divisible by n is a subgroup of $(\mathbb{Z}, +)$.
(ii) If A is an $m \times n$ matrix over $\mathbb{R}$, then $\{x \in \mathbb{R}^n : Ax = 0\}$ is a subgroup of $(\mathbb{R}^n, +)$.
(iii) $\{z \in \mathbb{C}^\times : |z| = 1\}$ and $\{e^{(1+i)t} : t \in \mathbb{R}\}$ are subgroups of $(\mathbb{C}^\times, \cdot)$ (where $\mathbb{C}^\times = \mathbb{C} \setminus \{0\}$). Draw these sets on an argand diagram.
(iv) For any $n \in \mathbb{N}$, the sets $\{g \in \mathrm{GL}(n, \mathbb{R}) : \mathrm{Det}(g) = 1\}$ and $\{g \in \mathrm{GL}(n, \mathbb{R}) : gg^T = I_n\}$ are subgroups of $\mathrm{GL}(n, \mathbb{R})$ (where g^T denotes the transpose of g and I_n is the $n \times n$ identity matrix).

Figure 1: Mathematical Task CS1E3

As the following discussion suggests, the problematic use of visual mediators is often irrelevant to students' object-level learning of the various group theoretic notions. The problematic use is often a result of inherited problems, in the form of metaphors from other mathematical discourses, such as Complex Analysis. Therefore, more often than not, the errors were located only on the Argand Diagram, in the context of an overall correct proof in which the algebraic reasoning and application of the governing metarules was often correct. Below, I list three representative examples.

Student A applied the routine and the governing metarules correctly in the second task in CS1E3iii, i.e. $\{e^{(1+i)t}: t \in R\}$, showing an overall complete grasp of the definitions of the involved notions and the applicability and closure conditions of the Subgroup Test. The only minor error was on the Argand Diagram. Student A presented it as if the spiral was starting from the origin, rather than approaching but never reaching it. This does not indicate an incomplete object-level learning of group theoretic notions, but rather a problematic metaphor from Complex Analysis.

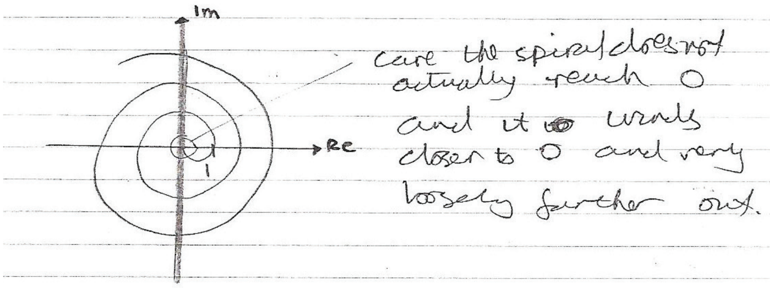
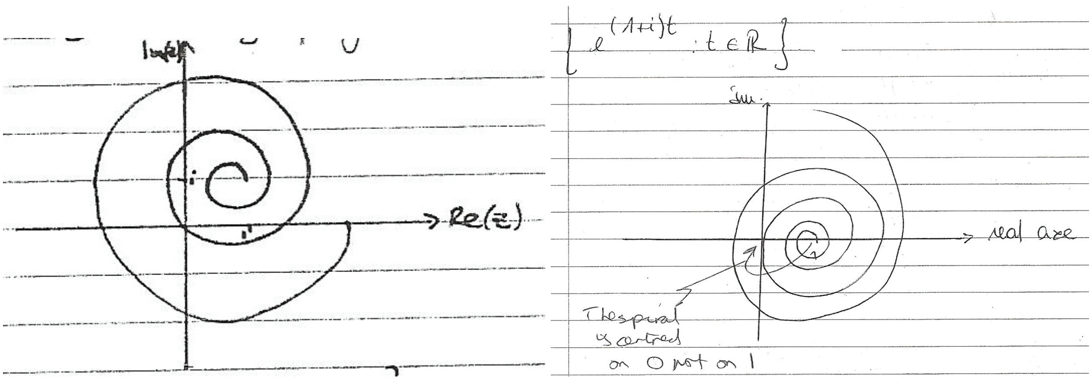


Figure 2: Part of Student A's solution

Her difficulty in drawing the Argand Diagram is obvious in the following excerpt, which reinforces the claim that her main problem was not on the learning of the notion of subgroup and the application of metarules, but rather on representing the subgroup on an Argand Diagram.

> Um yeah, they were alright, I find like – visualising sometimes the actual sets of them, quite difficult, to work out actually what you're talking about, and then – cos like they're quite big sets aren't they, like if you're doing an Argand Diagram, it's quite hard to prove it because you've got to do it for all of them, you can't just show the little ones, and then for part 3, we couldn't do them at all, cos we couldn't really show what they were… before we went to the seminars. And... Yeah, and I still can't draw that properly, you have to draw them don't you. Student A

Student B produced a spiral in the same exercise, but with its centre misplaced instead of the origin in $(1,1)$. Similarly, Student C misplaced the centre in $(1,0)$.



Figures 3 and 4: Parts of Students B and C's solutions

In both diagrams, there is no indication that the spiral approaches but never touches the centre. Again, this indicates problematic metaphors from Complex Analysis and consequently an undeveloped connectivity across the discourses of Group Theory and Complex Analysis. Therefore, in the learning of a new mathematical discourse, the discursive shift required is often disturbed by inherited problems that emerge through incomplete metaphors from other mathematical discourses. These problems need to be overcome, in order for the students to able to complete their learning in the new discourse. Another suggestion is that Argand Diagrams in the context of Group

Theory represent algebraic structures, and therefore are used in a different way then in Complex Analysis. In Group Theory, an Argand Diagram represents a group, whereas in complex analysis it represents a complex number of the form $z = x + iy$. This distinction between the two uses is probably causing a commognitive conflict, which causes further confusion, which needs to be overcome in the new discourse.

CONCLUSION

This study investigated the use of visual mediators, and in particular the use of Argand Diagrams, in the learning of the notion of subgroup and the application of Subgroup Test, in the context of undergraduate mathematics students' first encounter with Group Theory. Analysis suggests that the use of visual mediators is often problematic, not necessarily due to partial learning of the group theoretic notions. In fact, the reported errors are often irrelevant to the object-level learning of the group-theoretic notions. They rather emerge due to two reasons: first, the problematic metaphors, inherited from Complex Analysis; second, the commognitive conflicts caused by the different purpose of Argand Diagrams, in the two mathematical discourses. A future study of larger scale will focus on the use of visual mediators, during the entire learning experience of undergraduate mathematics students with Abstract Algebra.

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Using programming to improve problem solving ability in primary three mathematics

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Technology has been playing an increasing role in teaching and learning at all education levels. The same can be said about nurturing the problem solving abilities. In this paper we present the results of the research project on using visual computer programming to improve problem-solving skills, which in turn increases students ability in mathematics. The results clearly indicate that students who were involved in the project their performance in EGMA mathematics test increased and were more confident in class.

INTRODUCTION

The use of technology in teaching and learning abstract concepts in mathematics has been demonstrated to improve the understanding and problem solving ability of students. However, the rate at which technology is advancing has made the research and practice in integrating ICT in education to lag behind (Pooja & Muddgal, 2017). The situation is made worse by the fact that effective integration of technology in teaching and learning requires a delicate balance of the interplay between pedagogy, content and knowledge (Soury-Lavergne, Vale, Ferrara, Khairiree, & Ladel, 2017). However, the use of computer programming teaching and learning has been shown to be an important skill for the development of problem solving abilities in addition to logical reasoning (Kalelioglu & Gulbahar, 2014).

In this paper the results of the project we undertook in teaching visual computer programming to primary three girls in two primary schools in Njombe region are reported. The comparison of the EGMA test between baseline and endline show a marked improvement in their performance. There was a 55% improvement from baseline to endline assessment survey. The performance of primary three girls who volunteered to take part in this study is in line with the national EGMA performance.

METHODOLOGY

This research project was done in Mtwango Ward – Njombe region for five weeks, where in the first meeting and last meeting we administered the EGMA test. The number of participants who took part was thirty girls from two primary schools. These girls were in class three, taking part was voluntary. We engaged the participants in out of school hours. In the first week participants were introduced to a computer programming called Scratch. The approach we took in conducting these classes was as follows. First, we introduced the participants to the basics of programming, followed by installing Scratch mobile application in their mobile phones. Secondly, we worked on programming examples together in class. Thirdly, we gave group assignments and every twenty minutes gave feedback of the progress. It must be noted that participants were given the freedom to consult other groups. This process was repeated for the remaining four weeks.

In order to make sure learning did not end after the class, participants were given take home assignments and encouraged to share and discuss among themselves using mobile phones. Since the participation in the project was voluntary parents had to make commitment to allow participants to use mobile phone for at least half an hour a day. The participation of teacher's as stakeholders was sought through the school.

We used Scratch Programming language (https://scratch.mit.edu/), which is a visual programming language. This language allowed students to think of a problem at a high level of abstraction i.e., in terms of what they wish to achieve and not to worry about the low-level syntax programming details. Scratch provided a platform for participants to look at problems in many different ways. It allowed them to try given examples, experiment, and even make mistakes. The rationale for its choice was based on the fact that it is open source, easier to learn and has a large user base online.

The results of the research project were evaluated using the early grade mathematics assessment (EGMA) tool. There was a 55% improvement from baseline to endline assessment survey. While it is not possible at this stage to attribute all the girls better achievements in EGMA to problem solving skills they acquired by using Scratch, evidence is abound to show that the skills they gained in programming gave them confidence in the EGMA tool. Further research is on-going to inform policy and practice.

CONCLUSION

This study has implications in both policy and practice. It is clear from the study that programming can be used to nurture problem solving ability, which in turn gives confidence and improves performance in mathematics and other subjects. It is important to note that mobile phones were used in this study.

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Investigating student engagement in learning mathematics with GeoGebra in Rwanda

Alphonse Uworwabayeho and Hyacinthe Mushimiyimana

ABSTRACT
There is a general consensus among educators that student engagement in learning mathematics constitutes an important part of solution to the national and international decline observed at the level of students' mathematical competency in performing basic arithmetic, geometrical transformations and algebraic manipulations. In addition, the use of technology in general in the teaching and learning process is becoming growing at notable speed due to the vast availability of various devices. However effective integration of these new technology tools in mathematical activities towards enhancing students' learning outcomes stays challenging. This paper brings its contribution by discussing student engagement in learning Geometry in Rwandan classroom through exploiting GeoGebra software. This is a result from classroom observations of a mathematics teacher and her thirty two students (14-15 years old) when learning the theorem: "the angle at the center of a circle is twice the angle at the circumference". Findings reveal that GeoGebra engaged students cognitively towards conjecturing the theorem. This results from a more learner-centered teaching approach that provided students with opportunities to exploring GeoGebra while working on the theorem previously dissected into small steps.

Key words: GeoGebra, student engagement, angle, mathematics, cognitive.

1. INTRODUCTORY BACKGROUND

Rwanda faces the challenges and pressing duty of eradicating poverty, enhancing equity and expanding access to education without compromising quality, and placing the country on a path of sustainable growth and development (MINEDUC, 2009). Recognizing that the education sector is a key player in addressing these challenges, efforts have been put in place at policy and implementation levels to develop a robust education system characterized by provision of holistic educational benefits to its citizens. This is within the "Vision 2020" context which seeks to transform Rwanda into a middle-income country by the year 2020. This transformation will not be achieved unless Rwanda transforms from a subsistence agriculture economy to a knowledge-based society (*ibid*). It is in this perspective that Rwanda committed to achieve certain international development targets notably Education For All (EFA) by 2015. Rwanda registers a current enrolment rate of 96,9% (MINEDUC,2016). However, these quantitative achievements give rise to a new concern on the quality of education which gets even more under pressure as more children attend school. Among other significant reforms, Rwanda's education system is currently shifting from a knowledge-based curriculum to a competency-based curriculum. These changes intend to improve the quality of teaching by increasing students' competencies (knowledge, skills and attitudes). This requires teachers to embark on new teaching strategies and teacher development is seen as a means to improve teaching practice and therefore learning. Uworwabayeho (2009) presented a case study of two secondary school teachers, Laetitia and Isaie, who were beginning to use Geometer's Sketchpad in their teaching of mathematics in order to make their classes more leaner-centred. This initiative was taken in collaboration with academics from higher learning institution namely Kigali Institute of Education today known as University of Rwanda-College of Education. Since then, Laetitia continued to collaborate with the same institution with regard to integration of ICT in the teaching of mathematics. She is regularly participating to workshop organized by the institution for teachers who in their turn are called to implement acquired skills in their classes. So, Laetitia and Alphonse kept a close relationship with regard to teacher professional development. This paper draws on data collected from Laetitia' classes by Hyacinthe for her master's degree in education at the Aga Khan University-Institute for Educational Development (AKU-IED). For ethical purpose especially to get a research authorization from the Rwandan Ministry of Education, Hyacinthe was affiliated to UR-CE under Alphonse's supervision. Before joining AKU-IED, Hyacinthe was teaching mathematics and introduction to computer science at secondary school and used also to participate to above mentioned workshops where she met Laetitia.

Given this background, like Ampah-Mensah (2011) we ask ourselves this question: why children have to go to school or in general why do we go to school? The basic and simplistic answer will be that we go to school to be educated or to learn. But education or learning in the school system involves both learners and teachers who are exploiting different tools to achieve learning goals. It is not doubtful that the teacher is a key player in this process. At the present, theories on learning place learners at the centre of the learning while the teacher becomes facilitator of the learning. In particular, student engagement has become one of the key concern and key strategy for educational and social reform particularly in middle and high schools (Harris, 2008).On one hand, there are debates in the literature about defining student engagement. There are many categories of engagement such as academic, cognitive, intellectual, emotional, behavioral, social, psychological, to name few. Parsons and Taylor (2011) suggest that it is necessary that learners need to be functioning in all areas of engagement for successful learning to take place. On the other hand, the use of technology in teaching and learning process has hypothetically shown many advantages in enhancing student engagement (White, 2012) and encouraging discovery learning (Bennet, 1999). Lee (2011) argues that the use of information and communication technology (ICT) improves the curiosity of being in group work whereby every student learns more from her or his colleagues; thus persisting on the task. In particular, this environment improves middle school students' learning as they are inquisitive, interested, or inspired rather than being bored by teacher lecturing. Furthermore, this orients students in a room where they will be able to learn how to "communicate, collaborate, create, think critically and innovate" (UNESCO, 2010; p.11). But Tomei (2010) observes that the successful integration of ICT into the classroom will depend on the ability of teachers to structure the learning environment in non-traditional ways, to connect new technology with new pedagogy, to develop socially active classrooms, encouraging cooperative interaction, collaborative learning, and group work. In this way, the present study intends to answer the question: to what extent the participant teacher engage students in learning mathematics through exploring GeoGebra? To answer this question, we observed the teaching approaches used by the participant teacher in relation to student engagement; and how students were engaged in learning mathematics through exploring GeoGebra. Data analysis was guided by technological pedagogical content knowledge (TPCK) theory that enabled us understand to what extent the teacher was willing to explore GeoGebra for improving her classroom practices towards engaging students in learning mathematics. Findings may be used by other mathematics teachers for their continuous professional development or any other education stakeholders in charge of teacher professional development.

2. LITERATURE REVIEW

Mathematics syllabus for upper primary (Mineduc, 2015) argues that mathematics is an excellent vehicle for the development and improvement of a person's intellectual competence in logical reasoning, spatial visualisation, analysis and abstract thought. Learning mathematics develops numeracy, logical reasoning skills, critical thinking and problem solving skills. There is ample research evidence to the effect that the use of ICTs as teaching and learning tools brings about students' collaborative learning environment, thinking, visualizing, exploring relationships and patterns, students' motivation to learn, engagement in learning, and their independence in learning (Escuder & Furner, 2012; Sutherland, Robertson & John, 2008). However, this needs teachers who have enough skills which enable them to integrate new technology successfully. In practice, Uworwabayeho (2012) identified two opposite apprehensions from teachers. On one hand, the teacher appreciates the pedagogical benefit of using ICT tools such as GeoGebra in the teaching in the sense it allows students engagement in solving mathematical problems; thus cutting off habits of the teacher lecturing method. On the other hand, the teacher is afraid that students may not be able to transfer what they learnt using these technological tools to daily situations. In other words, students may rely on the feedback and facilities offered by the software without any effort to understand mathematical objects. This teacher's dilemma reflects the difficult transition from pedagogical content knowledge which is knowledge about how to make a subject understandable to learners to technological pedagogical content knowledge which includes knowledge about how technology may be used to provide new ways of teaching content (Niess, 2005).

The technological pedagogical content knowledge (TPCK) entails describing how teachers' understanding of technologies and pedagogical content knowledge interact with one another to produce effective teaching with technologies (Nguyen, 2014). An overview on TPCK components is provided by Koehler and Mishra (2008).

Based on the research question "to what extent the participant teacher engage students in learning mathematics through exploring GeoGebra?", the present study was limited on technological pedagogical knowledge (TPK). The TPK is an understanding of how teaching and learning changes when particular technologies are used. Paraphrasing Koehler and Mishra (2008,p.16), this includes knowing the pedagogical affordances and constraints of a range of GeoGebra tools as they relate to geometry discipline and developmentally appropriate pedagogical designs and strategies. It is assumed that participant teacher masters the mathematics content to be taught to her students. So the study focused on to what extent the teacher was using the technology for engaging students in their learning. Parsons and Taylor (2011) show that from many years ago students engagement helps disengaged and disadvantaged learners attain and contribute to learning outcomes, reduces classroom disruptions and discipline issues; and engages learners in their learning about learning which help them to become skilled lifelong learners as contrasted to well behaved, attentive students. The existing literature presents the concept of student engagement as multifaceted. Lippman and Rivers (2008) propose to break down engagement into three main types: cognitive, behavioral and emotional. Behavioral consists of the students' level of participation in studying related activities and their involvement in academic and learning tasks. Emotional engagement consists of the relationships the students have with their teachers and their peers. Rotgans and Schmidt (2011) consider cognitive engagement as the extent to which students independently search for information from different sources including internet, i.e., when students engage in self-initiated information-seeking behaviors. They go on to suggest that the level of autonomy is inherently related to an activity or task and largely determines the degree to which students engage cognitively with that activity or task. Relating to use of GeoGebra, this occurs when students develop the notion of dynamic geometry as an instrument by means of the interaction with it. In order to construct the knowledge, students experiment to find their own strategies, interact with both the teacher and peers for solving the problem. Based on a combination of TPK and student engagement framework, analyzing data consisted in identifying what strategies and teaching techniques addressing cognitive, psychological, behavioral, physical, and social factors which engaged students in their learning.

3. METHODOLOGY AND DATA

3.1 Research methodology

Based on Yin (1994)'s classification of case study, the present study is exploratory and descriptive studying individual teacher and learners (cases) in classroom in one school and providing narrative accounts of classroom extracts to gain a broader picture of how GeoGebra can be used for enhancing students' engagement in mathematics classroom. Researchers were interested in taking field notes in mathematics classrooms where GeoGebra was used as a teaching aid. The research setting was Year 3 (one class) of ordinary level in one Rwandan secondary school (students are 14-15 years old) during September-October 2015. As mentioned in introductory background, the mathematics teacher has close professional relationship with authors. The description of the school is provided in Uworwabayeho (2009). During the academic year 2015, the school has a population of 897 students and 33 teachers. It has about 200 computers dispatched in two different rooms thereafter called computer labs. Though the school is among the best equipped schools with computers, these are all the time exploited for teaching ICT skills. So there is no room for other subject teachers to exploit these facilities. For the purpose of the study, the school administration made a special arrangement in allowing the teacher to use one computer lab on Mondays and Thursdays for two teaching hours each day for a month. This means the students were using GeoGebra in mathematics classes for their first time.

In total five GeoGebra-assisted lessons in which one student was using a computer took place. First four lessons aimed at familiarizing students with GeoGebra features while the fifth concerned with the study of the theorem: "the angle at the center of a circle is twice the angle at the circumference."

The content of field notes included overall classroom arrangement, teacher's activities and students' activities. Moreover, the researcher was given opportunity to take photos, notes from discussions with the teacher and interactions with students which helped her to gather supplementary information (Robson, 2002). This information was mainly about research participants' views on challenges and benefits of integrating GeoGebra in the teaching and learning process. So, data were qualitative in nature and data analysis followed qualitative approach.

3.2 Data Presentation and Findings

The four first lessons were about introducing GeoGebra features through representing mathematical objects. Though the teacher used to select specific tasks to be accomplished by students, most often students went beyond what was requested.

Students were very engaged doing different mathematical tasks such as rotation, measuring angles, reflection, representing 3D geometrical figures, translation to mention but a few. Some students drew a circle then inside put triangle, reflection in the y-axis and rotation through 45^0, draw a circle put rectangle then do tangent using translation of triangle. The researcher asked the student (a girl) who was drawing in 3D where she learned what she was performing. She answered that she applied what she previously seen from a movie. It was amazing where you could see students physically demonstrating their emotion about their achievements in the class. During this phase, the teacher was from time to time asking some questions and helping students to move forward especially when they were unable to find out what GeoGebra tool to be used for accomplishing specific task. Though students were seated one per computer you could see that students were curious to see what the neighbor was doing and show to their colleagues what they did and even ask when stuck. This seems to suggest that with the use of GeoGebra pair or group discussions are automatically encouraged.

For the fifth lesson taught on 1st October 2015, the teacher set up a mathematical problem which students were instructed to solve through exploring GeoGebra. The problem was Microsoft word written and projected on the classroom white board.

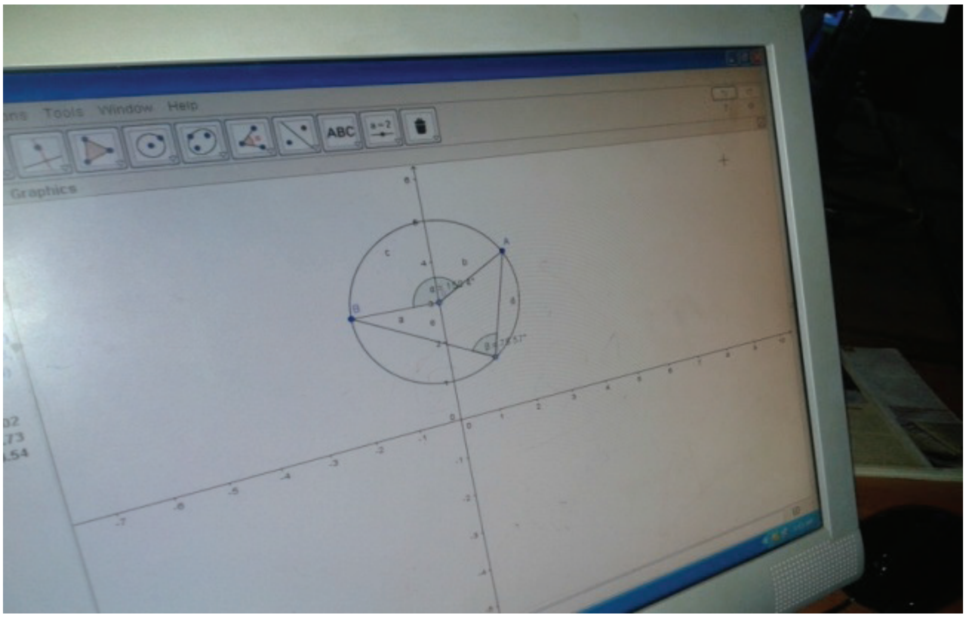
Learners' activity consists in the following:

- Using GeoGebra plot a circle with center 0
- On the circle place points A and B

- Join A to the center O and B to the center O
- Measure the angle at the center AOB
- Mark another point P on the same circle
- Join P with point A and B
- Measure the angle APB
- Compare angle AOB and angle APB
- What do you observe?
- What can you conclude?

The teacher dissects the theorem into small steps executed by students. To solve this problem, a student needs to interact with the mathematical task itself and the GeoGebra. In the previous lesson students were introduced to the main features (toolbox) of the GSP. Firstly, the student needs to engage with knowing commands for constructing the circle, points on circle, taking measurements; then the mathematical procedure for observing and concluding. In this case, GeoGebra can facilitate or constrain students in developing the new mathematical knowledge (hidden in the rubric, *what can you conclude*). In the former case, dynamic geometry can provide positive feedback by facilitating visualization. In the latter case, the student fails in constructing different mathematical involved in the problem such as points on circle and segment joining two points (antagonist context). Secondly, the student must deal with the mathematical constraints of: comparing two angles and concluding (cognitive context).

All students were engaged in working out the activity presented to them using GeoGebra. During this period the teacher was moving around answering students' questions whenever necessary. These questions were related to the mathematical concepts as well as GeoGebra features since students were curious to do more than what they were asked to do. It was observed that all students were able to use 'measurement tool' to measure angles AOB and APB and conclude that AOB was a double of APB. Dragging points A or B on the circle, one student, Charles say, observed that this characteristic was not changing.



Charles' work

This stage took about 50 minutes afterwards the teacher asked Charles to demonstrate what he did. Using a computer connected to the projector, Charles did step by step as indicated in the above exercise while others were following; then he concluded that the angle at the center is twice the angle at the circumference.

From discussions with the teacher, ICT in general and GeoGebra in particular is a teaching and learning tool enabling her to move from lecturing to towards more learner-centered teaching approach. When the teaching is assisted by GeoGebra students are engaged in doing activities and the teacher as she is moving along the class has more opportunities to support slow learners. When asked any relationship between teaching the lesson on 1st October 2015 (section 5.2.3) with chalk and board and GeoGebra, the teacher revealed: '*Using GeoGebra, you have all geometrical instruments and students are actively engaged while on the chalk and board I was doing myself and students were simply observing. During the teaching with ICT* [GeoGebra] *I was just asking students to explain what they were doing. The main difference about students' learning of mathematics is that to very large extent during the lesson they discover other things than those I asked them to do. Also teaching with ICT helps me too because when learners are busy working, I take a break.*' (Teacher's views after the lesson, 1st October 2015)

Asked any further positive element from using GeoGebra, the teacher revealed: 'a positive element is there, such as teaching from students' activities and conclusions. This reduces the teacher talk but requires her to assist students working on computer. In the teacher's words:

" *You have seen yourself that when students are working on computer, they interact with each other. During these interactions, one says to another whether this is true or not, and so on. When they don't come to a common understanding they call you and you can provide support." (*After the lesson, 1st October 2015*)*

4. DISCUSSIONS

Within the learner-centred model that enhances students' engagement in learning; teachers need to set a positive environment to enable students to construct their own knowledge. In other words, engaging students in learning may focus on strategies that allow students move beyond passive knowledge receivers to active knowledge constructors. However, as constructivists suggest, this student's construction of knowledge builds on what the student already knows. So, within this case study, one can conclude that the teacher structured her lesson in three stages: introduction, development and conclusion. During the introduction, the teacher engaged students in exploratory phase where students were asked to plot mathematical objects in GeoGebra and make some conclusions. In terms of TPCK, the teacher makes use of ICT as learning environment with a significant positive impact on the students' engagement. The teacher allowed her students to use their imagination and made up whatever they wanted so that they were almost making their learning. However, as the literature (e.g., Koehler & Mishra, 2008) suggests this is not enough, giving activities that allow students to make links to the real world, makes learning more relevant to them and more one so mathematics. In this line the teacher ended her teaching by requesting students to share their work to whole class as a way of providing feedback to their exploration. In doing so, the teacher was engaging students emotionally, cognitively and behaviorally in their learning.

In contrast to traditional lessons where a student who completes a given activity tends to be annoyed, students in GeoGebra assisted lessons were very motivated in plotting different geometrical figures but also doing mathematical transformations. Even if each student was using his computer, they were enthusiastic to ask or show their work with their neighbors. They were even not hesitating to ask the researcher some questions. In this way, the researcher also got opportunities to ask questions and noticing some specific events that were relevant to research questions. This shows to what extent the teaching approach enabled participant students' cognitive and emotional engagement. Furthermore, students affirm having understood some mathematical concepts which for them were still unclear but taken as guaranteed by the teacher. For example one student mentioned that the confusion he had about anticlockwise and clockwise was alleviated when he rotated a triangle in each of the two directions. Though the concept of 'direction' is taught from primary in the Rwandan mathematics curriculum, the student was still facing to a misunderstanding of the concept. Teaching the concept using a clock didn't lead the student as there was no way to change the direction of the 'aiguilles'. But GeoGebra offered this opportunity. He realized that clockwise and anticlockwise have different effects on the obtained triangle-image. In this way, the student was not only physically engaged but also academically, mentally and psychologically. Another found that measurement tool offered by GeoGebra makes things easier as he could take measurement of angles, length of rectangle side, etc. In particular, students pointed out that GeoGebra facilitates not only plotting 3D objects but also enables physical visualizations than when these objects are represented on paper or chalkboard.

All students affirm that use of GeoGebra increases their interest in conducting their own research and accuracy in drawing/calculations what and saves the time. It was observed that students went beyond teacher's expectations for example rotating, reflecting and plotting 3D objects. However, some students recognized that you do not know how to use the GeoGebra you may spend your time doing nothing important.

In their own words:

"There is nothing you can do in theory that you cannot do using GeoGebra but GeoGebra is so quick and makes concept understandable more than in the theory since GeoGebra allows us to see all the changes." (Charles, lesson taught on 1st October 2015).

It can be concluded that teaching mathematics with GeoGebra enabled students developing not only mathematical skills but also communication skills. Within this process students need to construct arguments for justifying their conclusions. In addition, students are listening to their colleague who is presenting her/his findings.

This engagement of students in mathematics conversations plays role to the development of students' skills and understanding. We can therefore infer that this teaching process encouraged students' behavioral and emotional engagement. Cabero (2011) argues that the use of ICT improves the curiosity of being in group work where every student needs to learn more from their colleagues by sharing in a student's forum collaboratively and individually. This increases thinking skills that are needed for problem solving (Smith & Broom, 2003). Example of lesson previously presented shows that GeoGebra engaged students in cooperative learning because teacher gave students time to discuss in their sits before one student shares his work with the whole class. This implies, all things being equal, even student who did not understand the activity on his/her own could understand it during either discussing in peer or the whole class sharing. In this way, the teacher used GeoGebra to enhance student engagement through performing mathematical activity, inquiring, exploring, evaluating, and experimenting, interacting with other students, and explaining their working.

CONCLUSION AND IMPLICATIONS

This paper argues that in order to engage her students in learning mathematics, the teacher gave students time to manipulate GeoGebra, then discuss in peers as they were seated and finally one student shared his/her work with the whole class. In this way, constructing mathematical objects using GeoGebra, conjecturing the theorem related to angles in circle, discussing with peers, presenting to the whole class or listening to the presenter provide to a learner different levels of autonomy in learning thus likely to result in cognitive engagement. Zepke and Leach (2011) revealed that the concept of "student engagement" is based on the belief that learning develops when learners are interested, questioning, or stimulated, and on the other hand learning be likely to suffer when learners are dispassionate, bored, disaffected, or otherwise "disengaged." In addition, observations of students during the teaching and learning process revealed students' emotions about their achievement/discovery of mathematical objects while using GeoGebra. Discussions with research participants indicated a positive overall perception of using GeoGebra in learning mathematics. Literature on integrating ICT in teaching and learning (e.g. Laborde, 2011) points out that the type of activity given to students constitute an important starting point for effective engagement of students in their learning and effective use of ICT tools. In this case study, it was observed GeoGebra enhancing students' cognitive engagement in the sense that students were invested in learning and in going beyond what was given by the teacher. Further analysis may focus on mathematical activities devolved to students towards students' engagement in their learning. Finally, if education stakeholders in Rwanda want to see ICT-assisted teaching and learning in practice as it is stated in the curriculum, this should be integral part of the school timetable as well teacher professional development.

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Sub-theme 5

Mathematics knowledge in and for teaching

Impact of using complexity science on capacity development for primary mathematics teaching in rural and remote communities

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From 2012 – 2018 Global Affairs Canada funded the University of Alberta and Brock University (Canada) to work in collaboration with the University of Dodoma (Tanzania) in a development project with a focus on building capacity for mathematics teaching and learning in rural and remote communities in four regions of Tanzania. Framed by understandings of complex learning systems (Davis & Simmt, 2003; 2006), the team focused on developing capacity for locally created and delivered ongoing professional learning in rural communities, raising awareness of the value of mathematics education and how members of the education sector (from teachers and school administrators to parents and ministry officials) play a role in supporting mathematics education, and making use of the resources that exist in communities, that are often thought of as resource challenged. In this paper, we describe the ways that complexity science perspectives contributed to the design and implementation of the project; and report on the early signs of project impact.

INTRODUCTION

The development project, known as *Hisabati ni Maisha* (mathematics is life/living), was built upon the lessons learned from an International Partnership Development research funded by the Social Sciences and Humanities Research Council of Canada (Simmt et al., 2014) and framed by understandings of complex learning systems (Davis & Simmt, 2003; 2006). There were three distinct, yet interconnected, hubs of activities in the design of the project: teacher education policy, teacher education, and community awareness to increase children's mathematical literacy through enhanced quality of mathematics teaching. The project worked in 9 districts (27 wards) in 4 regions in central Tanzania. In this paper, we describe the ways that complexity science perspectives contributed to the design and implementation of the project and report on the impact and continuity of project activities on primary school teachers, teacher educators, leaders, and community members in the four regions.

COMPLEXITY SCIENCE PERSPECTIVES

Complexity science is the study of complex systems; that is the study of systems with many parts that interact to produce a behavior that cannot easily be explained in terms of the sum of interactions between individual agents. Davis & Simmt (2003) described four features of complex systems in relation to mathematics classrooms: diversity ("variation among and mutability of its parts" (p. 148)), redundancy ("sameness among agents – in background, purpose, and so on" (p. 150)), decentralized control ("within a complex system, appropriate action can only be conditioned by external authorities, not imposed" (p. 153)), neighbouring interactions ("agents within a complex system must be able to share and affect one another's activities" (p. 155)). In this paper we describe the ways in which these features were used in the design and implementation of the project.

DESIGN AND IMPLEMENTATION OF THE PROJECT

Three distinct, yet interconnected, hubs of activities: teacher education policy, teacher education, and community awareness; were deliberately planned to operate on different aspects of the education enterprise to heighten the potential for project impact. Each hub is described below.

Teacher Education Policy Hub

The teacher education policy hub was designed to bring together diverse individuals with different educational experiences, in specialized roles (e.g. teacher college principal, district education officer, regional education officer), with diverse experiences in developing (ministry of education representatives) and implementing policies (regional education officer, representatives from Tanzania Institute of Education, zonal quality assurance officer, representatives of national level organizations such as the Tanzania Teachers Union and the Mathematics Association of Tanzania), with similar backgrounds (e.g. working as educational leaders within the education sector and all have mathematics teaching as a part of their responsibility) to engage in interactions about policy development and implementation that would support mathematics teaching in rural communities.

The activity in this hub consisted of 5 meetings over the duration of the project. The meetings focused on: an approach to developing policy from the grass roots, an analysis of gender inclusive policies and their implementation, and the development of policy recommendations. After two meetings we noticed that the participation level of some of the individuals within the activities were greatly influenced by the high level roles they held within the education sector; which meant that they could not participate fully. Reflecting on the situation the project team recognized that this hub was being compromised by the disruptions in the meetings and by the absence of critical persons. With diversity of agents and neighboring interactions critical to the ongoing maintenance of the group as a learning system there was a need to increase the number of stable agents (members that would participate fully and in each meeting). Meanwhile, focus group activities in which head teachers and ward education coordinators participated highlighted their roles in policy. Further, they did not have the demands on their positions that required them to respond to immediate issues as the high level government officials had. A decision was made to have 28 Head Teachers and 28 Ward Coordinators join this hub for the final 3 meetings. Their presence stabilized the group and resulted in effective and productive meetings that resulted in five policy briefs.

Teacher Education Hub

The activities within the teacher education hub focused on developing capacity for mathematics teacher education in rural communities. Three activities: short courses for district academic officers, district adult education officers, teacher college tutors, university lecturers, district quality assurance officers, and lead primary school teachers; graduate studies for 25 individuals: 22 teachers and tutors (master level), two university junior lecturers (PhD) and one government curriculum developer (PhD); and teacher professional development for 430 primary school teachers. Features of complexity unfolded in the design and implementation of the short courses.

The short courses involved 53 participants from 9 districts in Dodoma, Morogoro, Singida and Iringa regions over the 4 years of the project implementation. The 4.5 short courses, each two weeks in length, were structured around developing mathematical and pedagogical knowledge (topics such as number, ethnomathematics and problem solving, mathematical proficiency, assessment, using local resources, and gender strategies) as well as skills required for the sustainability of the initiatives (e.g. professional development models and proposal writing). Within the short courses participants from each district were asked to work together to ensure that there was diversity in the experiences and roles of the individuals. This diversity would result in expanding people’s awareness of the range of possibilities there were for understanding mathematics, teaching mathematics to teachers, resourcing mathematics classes, among other things. It also served to build a network of educators within a district who could support each other and teachers of mathematics. Control over various aspects of the activities and the project shifted among project coordinators and various groups of participants as decisions about implementation of the project activities were needed. Examples of this included: decisions as to who would develop and deliver PD modules for primary school teachers in each district; a participant list for the community workshops; development and implementation of funded micro-projects.

Community Awareness Hub

The focus of the activities in this hub was to raise awareness, through community workshops, of the value of mathematics education, gender, how members of the community (parents and families, religious leaders, business owners, peasants, nurses and doctors, and village leaders) play a role in supporting mathematics teaching in rural and remote communities, and demonstrate the importance of mathematics by focusing on financial literacy, especially as it relates to school and village budgets. Two community development workshops were planned and facilitated for each district by short course participants. 245 participants from 27 wards attend the workshops. Although this hub was the smallest part of the project in terms of overall resources and participant numbers, it has great potential to add to the sustainability of the project. Adding agents with different roles (specializations) in the community to the overall project serves to extend the possibility for neighboring interactions. In other words, more people have a role to play in supporting the mathematics education of children and youth in rural and remote communities of Tanzania through small but potentially impactful actions. For example, a nun reflected on how she now sees a way to encourage children to work on their mathematics, and a village businessman painted the project slogan on his cart, promoting the message *hisabati ni maisha*.

Early Signs of Impact

We identify early signs of the impact by sharing illustrative comments from participants.

At the end of the project all of the participants in the teacher education policy hub reported that they'd learned something new about ways to sustain the ideas generated by the project and indicated they had an awareness of how the developed policy briefs could be used in their practices. One participant articulated that they had learned, *how different groups were able to meet and implement the goals of the project without affecting their roles and responsibility. For example, despite the officers having their own plan of actions for their roles and responsibility, they managed to fulfil the goals of improving the quality of mathematics teaching and learning in schools*.

Participants in the short course activity of the teacher education hub reported that they learned about a number of inclusive and participatory strategies (such as group discussions, gallery walk, songs, role plays, poems, games, and using multiple representations in teaching mathematics), that they felt more comfortable using these strategies in their own practices, and that they learned about how to develop and facilitate professional learning. One short course participant commented about the way that the short courses were structured, *When I was told that "we will be learning together", I asked myself, how is it possible to sit and learn together in the same class having this diversity of people? … However, after starting learning, there were no differences, the facilitators focused on collective understanding regardless of the participants' backgrounds*.

Community participants reported that they learned about how each individual in a community has a role to play in children's growth and learning of mathematics, that they learned about the connection between mathematics and their daily life, and they learned about how to prepare budgets (Paslawski et al, 2018). One of the participants commented, *…I realised that I was not the only one who was not a teacher, it included different member of the community such as religion leaders, district councillors, village chairpersons, guardians/parents, local clan leaders, teachers and teacher educators. The introduction of the meeting participants encouraged me to go further and understand what mathematics is*.

Conclusion

It is far too early to determine the sustainability of the project within the wards, regions, districts, country, and institutions. However, the capacity for sustainability is evident: five policy briefs were developed and have the potential to impact future policy considerations at the national, regional, and local levels; representatives from 9 different categories of educational stakeholders participated in the development of those policy briefs; graduate degrees completed by Tanzanian teacher college tutors, university instructors, and teachers; short course participants able to take leadership in conversations about instructional strategies, mathematics content, and professional learning within the four regions; and community members aware of the ways in which they can support mathematics teaching and learning.

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Collaborative workshops for sustainable teacher development

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The African Institute for Mathematical Sciences Schools Enrichment Centre (AIMSSEC) has developed well-trialled teacher workshops for groups of teachers to use independently in their local areas, complementing AIMSSEC's professional development courses for primary and secondary teachers, subject advisers, and field trainers. Workshops are freely available with a linked App, and target active learning with meaning-making, particularly in contexts which are low-resource and large-class. They are designed to further develop teachers' mathematics and mathematics pedagogy knowledge and feed directly into related lesson approaches, supporting improved rates of transition into mathematics and science careers for a range of learners. I draw on participant teacher interviews and written accounts to report on evidence of the use and impact of these workshops and related app in South Africa and beyond, their affordances and constraints, and ask whether this might provide a widely applicable and sustainable model for mathematics teacher development.

BACKGROUND: AIMSSEC MATHEMATICAL THINKING WORKSHOPS

AIMSSEC has for some years worked in South Africa to strengthen the professional knowledge of practising teachers of school mathematics in previously disadvantaged rural areas, whose students showed low transition rates into mathematics and science careers. This work has been successfully transferred to several other sub-Saharan countries. Although teaching is largely by volunteers, funding face to face elements has of course proved a challenge. AIMSSEC provides in-service courses focused on both subject knowledge and subject pedagogical knowledge at several levels: a 3-month Mathematical Thinking (MT) course, endorsed with 15 professional development points by South African authorities; a 2-year level 6 Advanced Certificate in Education (ACE) course and a 2-year level 7 Advanced Diploma in Education (ADE). Each of these features residential elements so teachers experience the approaches being advocated, but also sustained periods where teachers experiment with, reflect on and evaluate those approaches for their learners, supported at a distance by in-country or international mathematics education experts through online forums, individual support, and sometimes, end of year examinations. Courses therefore adhere to our best understanding of characteristics of effective professional development (e.g. Desimone, 2009).

They are designed for the many teachers who are mathematically under-qualified in international terms, and prepare them to be more effective teachers, heads of departments and subject advisers. A particular aspect of AIMSSEC residential courses is coverage of key ICT skills so that teachers are more confident to harness freely available resources and software to support teaching and learning, if they have access to the Internet. To date, AIMSSEC has trained nearly 2000 teachers on the MT short course, while 215 teachers have graduated from the two year ACE course.

A key feature of ACE courses in particular is that teachers are equipped, and encouraged, to take back to their local areas materials and approaches similar to those experienced, and use them in facilitating local self-help teacher workshops. A range of such workshops for lower secondary students is represented in a book (Hopkins et al, 2016), but importantly, professional resources for workshops at all levels are freely available at https://aiminghigh.aimssec.ac.za/ or for any android smartphone the related free App can be downloaded from https://www.appszoom.com/android-app/aimssec-aiming-high-bidgro.html . Each workshop guide is aimed at groups of teachers working together, discussing approaches, deepening their understanding of the related mathematics, and engaging in strategies for active approaches to teaching and learning mathematics with meaning, often through problem-solving and guided re-invention. Guides use practical approaches to learning, and support development of a range of classroom communication – by teacher and learners, pointing too to further evidence-based reading. All the approaches have been iteratively developed through extensive trials with teachers in low-resource, large-class contexts. Each provides a summary of a mathematical topic, activities to work through with colleagues in a teacher workshop, lesson activities and suggestions for teaching, advice for implementing teaching strategies, and additional resources such as worksheets and templates. Importantly, by understanding why the suggested approaches work, and experiencing that for themselves, teachers are empowered to apply them to other areas of the curriculum.

STUDY

This paper draws on 6 short (250-350 word) unstructured accounts of teachers' journeys with AIMSSEC: the total provided in response to an open call after one ACE course.

Teacher	Phase and current role	AIMSSEC background
T1	Lower secondary (grade 7-10), local leader of courses for 60 schools (2 groups)	MT, ACE
T2	Lower secondary (grade 7-10), local leader of courses using the MT book. Now Head of Department.	MT, ACE
T3	Lower secondary (grade 7-9). Now Headteacher	MT, ACE, teaching assistant, MT course

		tutor
T4	Upper Secondary (grade 10-12). Now working on national undergraduate mathematics scholarship scheme but still teaching in schools every week	MT, ACE
T5	Primary (grade 4-6)	MT, ACE, teaching assistant, MT course tutor
T6	Lower secondary (grade 7-9). After ACE, enrolled for BEd in Mathematics, then to a Masters in Mathematics Education: wants to move into teacher education	MT, ACE

Table 2: **Study participants**

These accounts were analysed via grounded thematic coding (Charmaz, 2006). Because the accounts were short, and written in English, I argue they represent key points in teachers' thinking about AIMSSEC. These fell into four categories, exemplified in every free response given: teachers' conceptions of mathematics, of teaching, wider impact of their experience on other teachers and on learners, and their affective responses.

The paper also captures the reflections of teacher 1 in a recent semi-structured interview which asked about perceived impact, and the role of face to face AIMSSEC sessions within that. Critically, it probed perceptions about the potential of the App to support significant teacher development along AIMSSEC-adopted principles, without any local facilitator attending a face to face AIMSSEC course. The teacher professional development literature is clear that experiencing the target approaches is important, as is expert support and challenge (e.g. Desimone, 2009), but the question remains, how effective is it to do so remotely, albeit if then embodied in structured teacher workshop activity and reflection – especially if such resources are not in teachers' home language?

FINDINGS AND DISCUSSION

Teachers conceptions of mathematics

Teachers' evaluations of MT courses almost universally show such change, for example, 'I discovered that mathematics is not only about numbers, it is a language in itself. Expressions and equations are a short way of narrating a story, they are not just numbers and symbols without meaning' (Hopkins et al 2016, p204). However, the responses of these ACE graduates were striking:

> I experienced a "mathematics-culture-shock", as the course exposed my shallow understanding of mathematical concepts…This course changed my life. For the first time,

> I discovered that mathematical concepts can be taught as objects. I reflected on my teaching and realized I had not been doing justice to the learners, and neither had my schoolteachers to me. Apart from using teacher-centred approaches and forcing learning to stick to formulae, I was indirectly excluding learners from learning and enjoying mathematics (T4)

> I shared the strategies I learnt at AIMSSEC with teachers from the two circuits. I always pointed out that, "Mathematics is not about formulae; it is about relating concepts for better understanding." ' (T1)

Teachers conceptions of mathematics teaching

Teachers referred to changes in the ways they thought about teaching mathematics: 'it was a wonderful journey of exploring and learning how to teach mathematics... we were always actively involved in the learning process' (T6); 'it helped me to understand different teaching methods I could use in my classroom to make the learners understand mathematics better' (T2); The (MT) 10 days changed my outlook and teaching approach and opened my eyes to many new experiences (T5).

Wider impact of teachers' experience on other teachers and on learners

Teachers talked largely about impact on other teachers, but sometimes pointed to specific impact on learners: '(I) share the knowledge I have gained with other teachers in my school for application in the different learning areas (T2);

> My colleagues were amazed these workshops changed our approach to teaching and to mathematics. Instructions were simple, clear, understandable and straightforward. They were thought-provoking and challenging to people's creativity – and learners enjoy them and learn with understanding… if AIMSSEC had the chance to teach in our schools, every learner would be passing mathematics: many more of our students are'. (T4)

> The skills I have gained will continue to make a difference… within and outside my school and community. Since my involvement with AIMSSEC, our mathematics pass results have increased by 25%: learners are achieving that with understanding and choosing to study more mathematics. (T3)

> The skills I learnt from the AIMSSEC courses boosted my confidence in presenting workshops for educators and I began … mentoring sessions in my district, piloting with 2 circuits, each with 30 schools, and I also conducted Grade 9 Spring School for the pilot circuits. (T1)

> AIMSSEC teaching strategies have inspired me to further my studies in mathematics and to encourage capacity development of fellow South Africans. I am continually broadening my horizons and this encourages me to add value in mathematics education in my province, the country and in Africa as a continent. (T1)

Teacher 1 was able to articulate a sequence of impacts she had observed in her district, from herself to other teachers to learners, citing observations of classrooms 'where teachers and learners are more confident and engaged, learners want to discuss the meaning of what they are doing, can justify that and support one another in thinking mathematically'. She also pointed to improved examination results and greater interest and success in pursuing mathematics-dependent study further.

Teachers' affective responses

Teachers talked of a new interest, commitment and enthusiasm- but also of empowerment that follows from greater understanding of the mathematics and of teaching and learning: 'I can empower teachers who in turn will go back to their respective schools and empower thousands of learners in mathematics' (T6); I became more interested in teaching the subject (T2);

> I returned to …, a "new" teacher, one who turned mathematics lessons into joyful learning sessions. The ACE course was another extremely enjoyable and deeply engaging course. In addition to the enriching content… It made me want to give of my very best; proving, once again, what an amazing impact a motivated teacher can have …I think that teachers need to learn and teach more than just content alone. They need to teach hope and bring joy to the classroom. That comes with confidence and genuine self-enjoyment. (T5)

Teacher 1 talked about the 'excitement of learning and doing mathematics together in new ways', but also of a need for confidence to lead that. She had used the App to access additional workshops but was doubtful an App standing alone could endow that confidence – or the in-depth understanding of why workshops were so constructed. Research is needed to find out how much expert-supported face to face contact is needed to support valid local enactment of workshops – or whether such support could perhaps be effective remotely, but e.g. live online: such considerations are key to affordable and sustainable effective development of teachers of mathematics in Africa and elsewhere.

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Mathematics knowledge in and for teaching

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Abstract

Teachers' mathematical knowledge is critical to effective instruction as well as developing competent and skilled individuals in mathematics, the basis of survival in the 21st century which is an information age. It is not possible to live in the modern world without making some use of mathematics. It should as well be noted that many developments and decisions made in industry, businesses, social services, government policy and planning and so on, rely to some extent on the use of mathematics. It follows from this view that all teachers of mathematics need quality mathematics education that will enable them help individuals they teach to live a full life in the society, if they are to contribute towards the prosperity of their nations. Thinking about Quality mathematics education implies reflection on mathematics knowledge required in teaching and mathematics knowledge for teaching. Teachers who possess mathematics knowledge in teaching and for teaching have the potential to develop problem-solving skills among their students, a hub to innovation. It therefore follows that teachers ought to be well equipped with the mathematics knowledge to apply in their classrooms as well as mathematics knowledge to deliver to their students so as to develop their power to apply mathematics in solving problems in life situations. Hence we shall need to deliberately discuss the two concepts as teachers and apply them henceforth. The paper will therefore explain the concept of mathematics knowledge in and for teaching, aimed at equipping individuals with quality mathematics. The implication of the mathematics knowledge in and for teaching to teachers and teacher educators are identified. Finally, a conclusion as well as recommendations to teachers is provided.

Mathematics knowledge in teaching

Mathematics teachers require that they are well grounded in the pedagogy of teaching mathematics in order to enable students not only to succeed in school but also to be successful in life after school. This is because mathematics is one of the essential subjects that allow us to function in the world.

Many teachers of mathematics consciously or unconsciously apply mathematics knowledge in teaching in a number of ways. Such teachers do not only deliver mathematical content to their students, they also know the pedagogy that determines how their students carefully learn; recognize that in order for students to effectively use mathematics, they need to understand the concepts presented as well as become fluent with the skill taught.

They know what the students need to learn; know what they already know; encourage risk taking; create purposeful learning experiences; create thought-provoking tasks that challenge students to think creatively and critically. Such teachers realize the paradigm today that focuses on students achieving school and career readiness in life

beyond school, unlike the old one that focused on enabling students to achieve success at school only.
In addition, teachers who apply mathematical knowledge in teaching recognize the importance of using concrete materials and experiences as well as visual representations to develop a deep understanding of the subject. They use a variety of methods of instruction that are manipulative, thought provoking according to students' needs and abilities, building and maintaining their curiosity and interest.

Mathematical knowledge for teaching
Effective mathematics teachers appropriately teach students from the perspective of helping them to learn it. They are ready to teach students a mathematical idea, method or other aspect. Ball, D. (2011) suggests that teaching mathematics is a special kind of mathematical work that includes solving special kinds of mathematics problems, engaging in specialized mathematical reasoning and use of mathematical language in careful ways.
The mathematical knowledge for teaching also comprises of various domains. In other words, it is multi-dimensional in nature. It includes knowledge of various mathematical concepts (content knowledge for example; number & operations, algebra, geometry, statistics and so on); pedagogical content knowledge; knowledge of the curriculum as well as knowledge of content and students.
Content knowledge (knowing what to teach) is seen as a necessary precondition for successful teaching (Ball et al. 200; Shulman 1986, 1987). Nevertheless, most empirical research on
instruction does not show this relationship, which may be due to the fact that pedagogy cannot be used to investigate the influence of teachers' content knowledge on instruction or even students' learning outcomes. Unless the two domains are applied together as one entity which demands for those skilled mathematics teachers in both aspects.

Ball, D. L., Thames, M. H., & Phelps, G. (2008) in their research on "Content knowledge for teaching" indicate at least two empirically discernable subdomains within pedagogical content knowledge (knowledge of content and students and knowledge of content and teaching) and an important subdomain of "pure" content knowledge unique to the work of teaching, specialized content knowledge, which is distinct from the common content knowledge needed by teachers and non-teachers alike. All the subdomains are equally important hence they need to be blended and not used in isolation in order to help learners view mathematics functional in real life situations, thus impact their socio-economic life in society functionally.
Pedagogical content knowledge, according to Shulman, (1986) is a type of knowledge that is unique to teachers, and is based on the manner in which teachers relate their pedagogical knowledge (what they know about teaching) to their subject matter knowledge (what they know about what they teach). It therefore integrates the teachers' pedagogical knowledge and their subject matter knowledge. It is a form of knowledge that makes mathematics teachers rather than mathematicians. They organize the mathematics content from a teaching perspective focused on helping students to understand specific mathematics concepts.

Conclusion

In conclusion, it is very clear that both content and pedagogical knowledge are actually indistinguishable body of understanding as we think about the math teachers and their work. Since teachers with knowledge in teaching may not necessarily have the knowledge for teaching, yet teachers with knowledge for teaching may deliberately imply they have pedagogical content knowledge, it follows that mathematics teachers ought to be prepared to effectively and skillfully integrate all the domains of understanding, that is they should be in position to apply the Technological Pedagogical Content Knowledge (TPCK) in the teaching of mathematics. Hence students will get excited about mathematics and will significantly achieve in school and after school career.

Recommendations for Teachers

- Mathematics teachers need to more often reflect on or think about why they teach specific ideas the way they do.
- Teachers need also to know much more about teaching subject matter concepts to students than they are aware. This is pedagogical content knowledge that teachers ought to think about as important because it determines what a teacher does from time to time in the classroom, as well as influencing long term planning.
- Teachers need to explore how students think about the mathematics concepts being taught. Help students to relate the learnt mathematics concept to real life situations.
- It is a critical practice sharing and discussing with other teachers, were teachers exchange instructional strategies, how to teach difficult concepts or dealing with specific/diverse learners.
- Teachers should also carry out action research to help them identify the good practices that are contributing to intended learning outcomes or the weaknesses and/or errors that need to be eliminated to improve results from a learning program.
- Mathematics education teachers need to carry out intensive reading of relevant text books/e-learning and make use of the material learnt in life situations.
- It requires that mathematics teachers in addition to pedagogical content knowledge, also most importantly apply technology (Technological Pedagogical Content Knowledge TPCK) to be able to produce persons that will meet the demands of the 21st century.

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An examination of content knowledge for developing geometric proofs

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This paper presents findings from a qualitative case study which explored content knowledge for developing geometric proofs. The study addressed the question; what characteristics are constituted in geometric proofs developed by Malawian secondary mathematics teachers? Data was generated by administering pencil and paper test to four Malawian secondary mathematics teachers who were selected purposefully. The key terms in the definition of mathematical proof by Stylianides and Ball were used as pre-determined themes to analyse the proofs. The findings show that in addition to deductive reasoning, geometric proof development also requires knowledge of identifying and developing an in-between proof. Furthermore, development of a correct in-between proof requires knowledge of making proper representation of a word problem into a diagram and knowledge of interacting with the diagram using both deductive and geometric reasoning which can be called geometric-deductive reasoning.

BACKGROUND

Proving is a vehicle for deep learning in all mathematics content areas (Stylianides & Ball, 2008). In Malawian mathematics curriculum, geometry is the main area of mathematics which offers students many opportunities for understanding proof development. However, most Malawian students fail to develop a geometric proofs during national examinations (Malawi National Examinations Board [MANEB], 2013). The mistakes that the students make when developing geometric proofs during national examinations show that the students do not understand how to develop valid and logical geometric statements (MANEB, 2013). The problem of students' challenges in developing geometric proofs is also reported by several researchers (Battista, 2007; Chinnappan, Ekanayake & Brown, 2012). Several reasons have been advanced for this problem; students' poor background of geometry, interwoven nature of geometry and geometric proving, poor teaching strategies which emphasise on memorising the proof steps and deductive reasoning, and lack of research focusing on the concept of proof development in the study of Geometry (Battista, 2007; Jones et al., 2009; Usiskin, Peressini, Marchisotto & Stanley, 2003). There is a considerable amount of research that has been conducted with an aim of developing frameworks and models of pedagogy for teaching deductive reasoning in older to improve students' ability to develop geometric proofs (Herbst & Brach, 2006; Jones et al., 2009). Knowledge of deductive reasoning has been emphasised in research because it is assumed as the main source of geometric proving due to its contribution to logical reasoning (Herbst & Brach, 2006). Despite these studies, geometric proof development continues to be challenging for many students in the world (Chinnappan et al., 2012). This might be because the studies have not focused on understanding knowledge constituted in geometric proofs despite Battista's (2007) suggestion to focus on content knowledge used in geometric proof development. This study argues that knowledge of deductive reasoning alone is not enough for geometric proof development, hence the need to examine other knowledge components required for geometric proof development. The research question addressed in this paper is as follows: what characteristics of content knowledge are constituted in geometric proofs developed by Malawian teachers?

THEORETICAL FRAMEWORK AND METHODOLOGY

Stylianides and Ball (2008) define proof as a mathematical argument that fulfills three criteria; (i) it uses set of accepted statements that are true and available without further justification; (ii) it employs valid modes of argumentation; and (iii) it is communicated with appropriate modes of argument representation. Set of accepted statements means that the proof uses statements and definitions that are mathematically true, valid modes of argumentation means applying correct reasons for a proposition, and appropriate argument representation means linking definitions, axioms and theorems in a deductive manner (Stylianides & Ball, 2008). This means that the terms true, valid and appropriate relate to the content of mathematical proofs which is the focus of this paper. I used Stylianides and Ball's (2008) criteria as a framework for analysing geometric proofs developed by secondary school Malawian teachers in order to examine content knowledge required for geometric proof development. The findings reported in this paper are part of the ongoing qualitative case study which aims at exploring knowledge for teaching geometric proofs. Data analysed in this paper was collected through purposeful sampling. Four secondary mathematics teachers who have been teaching secondary mathematics for at least 4 years were selected to participate in the study with an assumption that they would provide rich data gained from their long experience. The teachers proved 2 geometric proof problems which were performed poorly by form four (grade 12) students at Malawian National Examination in 2013.

FINDINGS AND DISCUSSION

In this paper, only findings on one of the geometric proof problems are presented. The problem is as follows: AB is the diameter of a circle with centre O and AC is a chord. OD is perpendicular to AC. Prove that BC is two times OD. The teachers developed several proofs in response to the problem. There were three main approaches that the teachers used to develop proofs for the problem. The first approach involved the use of similarity theorem, second approach involved the use of rectangle properties, and the third approach involved the use of Pythagoras theorem. The statements that were developed by the teachers using the first and second approaches were similar. For the sake of space, I will only focus on presenting examples of proofs that were developed using first and second approach. Figure 1 presents a proof which was developed by teacher 1 (T1) using similarity theorem.

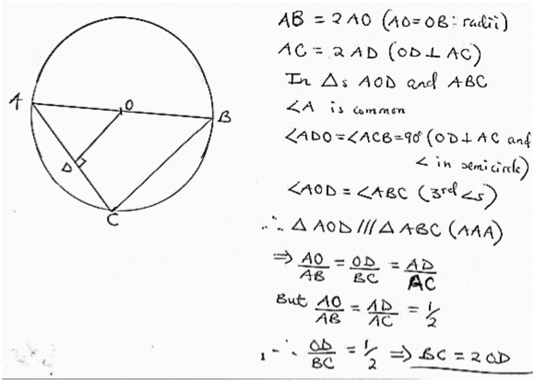

Figure 1: proof developed by T1

Figure 1 shows that T1 started the process of proving by representing the word problem into a diagram before developing the proving statements. The figure also shows that T1 started by developing proving statements which would be used to develop other proving statements. For example, proving statements 1 (AB = 2AO) and 2 (AC = 2AD) have been used to show that triagles AOD and ABC are similar. This shows that there is logical reasoning in the proof developed by T1. As such the proof agrees with Herbst and Brach (2006) that knowledge of deductive reasoning is the main source of geometric proving because of its contribution to logical reasoning. Proving statements 1 and 2 are valid because they have used correct geometric properties of radii and theorem of angle property of the circle. Similarly, the proving statements for the similarity proof are also valid because they have used valid mode of argument representation. The proof shows that T1 applied several geometric properties in coming up with the connected proving statements; radii, perpendicular bisector, angle in a semicircle. This implies that each of the proving statements was developed through geometric reasoning which involved interacting with the diagram and relating it to geometric properties (Herbst, & Brach, 2006). This shows that in geometric proof development, deductive reasoning requires knowledge of geometric reasoning. The similarity proof is a valid mode of argument representation because it has been used correctly to establish the proportionality of the corresponding lines in the triangles AOD and ABC with an aim of providing logical connection between BC and OD. This suggest that the in-between proof (similarity) was necessary for deductive reasoning. The same observation is also made in the following proof developed by teacher 2 (T2) using the second approach.

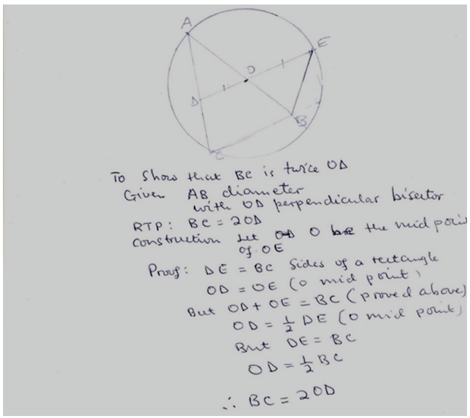

Figure 2: proof constructed by T2

T2 also started by representing the word problem into the diagram (figure 2), and then he added a construction to the diagram by producing DO to E. Some of the information from the word problem has not been represented properly in the diagram drawn by T2. AB is not represented as the diameter of the circle because despite passing through the centre of the circle, one end of line AB is not touching the circumference, meaning that AB is not a chord, hence not a diameter as well. This might be the reason why T2 could not use the concepts of angle in a semi-circle as done by T1. This implies that geometric proof development also requires knowledge of proper representation of a word problem into a diagram. As Usiskin et al. (2003) pointed out, although diagrams are often useful in proofs, when not drawn carefully and correctly, they can lead to invalid assumptions and false conclusions. This is evidenced in the the first proving statement developed by T2 which reads that *DE = BC (sides of a rectangle)*. The statement implies that diagram BCDE is a rectangle, a fact which is neither given in the problem nor proved in the proof. As such the statement that DE = BC is not valid because it requires further justification. T2 was supposed to start by developing an in-between proof to show that DEBC is a rectangle before applying the rectangle properties. The lack of an in-between proof for showing that OECD is a rectangle indicate that there is a gap in argument representation, hence the proof has not used appropriate arguement representation (Stylianides & Ball, 2008). The second proving statement is that OD = OE (O mid-point). However, the diagram shows that OE is radius of the circle because it is drawn from the centre to the circunference, while OD is less than the radius because it does not touch the circunference of the circle. Therefore the statement that OD = CE is not valid because it does not contain true mathematical statements. Although the proof developed by T2 shows that the geometric statements are logically connected, a sign that deductive reasoning was applied in coming up with the statements, the proof is not correct because it has not used valid statements and valid modes of argument representation.

CONCLUSION

The findings suggest that in addition to knowledge of deductive reasoning, geometric proof development also requires knowledge of identifying and developing a correct in-between proof which can properly link the given information and the conclusion. The findings also show that development of a correct in-between proof requires knowledge of making proper representation of a word problem into a diagram and knowledge of interacting with the diagram using both deductive and geometric reasoning which can be called geometric-deductive reasoning.

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Teachers' pedagogical content knowledge and the teaching of statistics in secondary schols wakiso district in Uganda

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Abstract
Statistics is one of the most dynamic, rapidly growing and highly pertinent disciplines today. This study investigated teachers' pedagogical content knowledge (PCK) and the teaching of statistics in secondary schools in Wakiso District in Uganda. PCK was contextualized as teachers' general knowledge, pedagogical knowledge (PK) and content knowledge (CK). The sample of the study consisted of 60 secondary school mathematics (statistics) teachers. Data sources were the self-administered questionnaire, document analysis (schemes of work and lesson plans), classroom observations and post-lesson interviews. Quantitative data was analyzed using descriptive statistics and Pearson's Linear Correlation Coefficients. Qualitative data was analyzed using categorization and narratives. The findings of this study showed that there was no relationship between the teachers' PCK and the teaching of statistics. Mathematics teachers teach statistics theoretically and the implication for practice is that a lot of training has to equip them with statistical knowledge. It was recommended that Ministry of Education, Science, Technology and Sports (MoESTS), Uganda National Examinations Board (UNEB), National Curriculum Development Centre (NCDC), teachers and researchers collaborate together to uplift the standards of teaching of statistics in the country.

Introduction

Statistics is one of the most dynamic, rapidly growing and highly pertinent disciplines today. The knowledge of statistics provides a description of a summary of the past, the present and an approximation of the future for, say, an institution. In Uganda, statistics forms part of the mathematics course in the majority of schools at all age levels, ranging from primary to tertiary levels. At primary level, mathematics is taken as a single subject and at the lower secondary level it is taken as two separate options namely Mathematics 456 and Mathematics 475. Statistics then stands as a subject in its own right at the advanced secondary and tertiary levels. Whereas the Uganda National Examinations Board (UNEB) syllabus (UNEB, 2005, p.106) argues that at ordinary level ("O" level), "mathematics should be visualized as a vehicle for aiding a student to think, reason and articulate logically", statistics does not stand out as a subject in its own right but is rather embedded in the miscellaneous applications.

In the preparation of "O" level and advanced level ("A" level) syllabuses in Uganda by UNEB, the difficulty of some of the statistical concepts which these syllabi contain appears to be under-estimated, with the result that their statistical content is too extensive (UNEB, 2005). Consequently, these syllabi suffer the lack of emphasis on the use of real data collection and analysis by students, which is thus far from the call by UNEB to use sound statistical reasoning (Opolot-Okurut, Opyene-Eluk, &

Mwanamoiza, 2008). Further, the UNEB syllabi for both "O" and "A" levels (UNEB, 2007) make explicit statements on what statistical ideas and techniques ought to be taught in schools but unfortunately, there is no emphasis on the application of these techniques to understand and to interpret data in real-life situations. Learners commonly attempt questions in statistics but fail to score highly (UNEB, 2012), making the current status of teaching statistics far from satisfactory. Such reports make one wonder that if UNEB has set out its expectations for the teachers and assumedly teachers are taking up their roles, why then the failure in statistics by the students? According to McDuffie (2004), teachers need to have the knowledge of statistics in order to teach statistics and hence, teacher knowledge is fundamental to quality instruction.

Shulman (1986), outlined seven types of teacher knowledge for successful teaching. These include: "(i) content knowledge; (ii) general pedagogical knowledge; (iii) curriculum knowledge; (iv) pedagogical content knowledge (PCK); (v) knowledge of learners and their characteristics; (vi) knowledge of educational contexts; and (vii) knowledge of educational ends, purposes and values", (p. 8). Hence, he articulated the importance of knowledge to a teacher. Shulman stressed the importance of CK and PK to the teacher. He defined CK as "the amount and organization of knowledge per se in the mind of the teacher" (p. 9). Shulman defined PK as the knowledge of "how to teach" (p. 6), and more solidly later as, "how teachers manage classrooms, organize activities, allocate time and turns, structure assignments, ascribe praise and blame, formulate the levels of their questions, plan lessons, and judge general student understanding" (p. 8).

Shulman (1986) noted that CK and PK were inseparable, stressing that, "mere content knowledge [CK] is likely to be as useless pedagogically as content-free skill [i.e. PK]" (p. 8). Hence he proposed "pedagogical content knowledge" (PCK) for which he defined as, "pedagogical knowledge [PK], which goes beyond knowledge of subject matter per se [i.e. CK] to the dimension of subject matter knowledge for teaching [i.e. PK]" (p. 9). While there could be several contributory factors to students' poor performance in statistics, the teachers' PCK may have played a major role. Hence the need for this study to find the relationship between the teachers' levels of PCK and their teaching of statistics. In particular this study sought to find the relationship between the teachers' levels of general knowledge; pedagogical knowledge (PK); and content knowledge (CK) and the teaching of statistics in secondary schools.

Review of Related literature

Recent research on mathematics knowledge for teaching has focused on questions such as how mathematics knowledge is used in teaching. Many researchers (Callingham, Carmichael & Watson, 2015; Estrella, Olfos & Mena-Lorca, 2015; Ijeh, 2013; Ijeh & Onwu, 2012; Ma'rufi, Budayasa & Juniati, 2018; Rosenkranzer, Horsch, Schuler & Riess, 2017; Watson & Callingham, 2013) have examined teachers' knowledge and the roles knowledge plays in shaping teaching practices. For example, Callingham *et al*. (2015) explored the influence of teachers' PCK on explaining student achievement in statistics. They collected data from 789 students from three Australian states. The students completed three tests and their teachers completed a survey that included items measuring their PCK for teaching statistics. Through multilevel modelling of their data, they indicated that students' outcomes were influenced positively by their teacher's PCK. However, despite Callingham *et al*. developing valid instruments to measure both student and teacher CK and teachers' PCK, they found linking teachers' knowledge directly to students' learning outcomes elusive.

Estrella *et al*. (2015) developed a questionnaire on the knowledge of statistics among primary school teachers and their knowledge of how to teach statistics. Their questionnaire focused on teacher awareness of student statistical knowledge and the teaching of statistical content. The questionnaire comprised a total of 14 items and was administered to 85 primary school teachers and their 994 respective students in Chilean schools. Having performed exploratory and confirmatory factor analyses, they found the instrument to be valid and reliable. Ijeh (2013) studied the mathematics teachers' PCK for teaching statistics. He collected data from four mathematics teachers in a South African university through a conceptual knowledge exercise, concept mapping, lesson observation, questionnaire, video recording, teachers' written reports and document analysis. Having identified themes and patterns basing on iterative coding and categorisation of responses and observations, he found that mathematics teachers possessed topic-specific subject matter CK and conceptual knowledge to teach statistics in school mathematics.

Ijeh and Onwu (2012) studied how mathematics teachers applied PCK to identify and deal with learners' conceptions in statistics. They collected data about learners' conceptions (preconceptions and misconceptions) from four teachers in a South African university by means of lesson observation, teacher interview, questionnaires, video recordings, teachers' written reports and document analysis. Through coding and categorisation, they found that mathematics teachers deal with learners' conceptions in statistics by applying topic-specific instructional skills and strategies; using learners' responses to oral questioning; setting pre-activities; the checking and marking of learners' class work, homework and assignments; and analysing learners' responses to class work, homework and post-teaching discussions.

Ma'rufi *et al*. (2018) aimed at describing the profile of high school teachers' PCK in learning mathematics from the perspective of teaching experience. The focus of their research was how the novice teachers' PCK dealt with the knowledge of student. They defined the knowledge of student as a teacher's knowledge about the students' conception and misconception on limit of function material and the teacher's ability to cope with students' difficulties, mistakes, and misconceptions. They collected data from two high school mathematics teachers who had different teaching experiences through interview and observation. These two teachers were from the same high school in Indonesia. Having analysed their data qualitatively, Ma'rufi *et al*. showed that novice teachers' ability in analysing the cause of students' difficulties, mistakes, and misconceptions was limited. Hence, novice teachers tended to overcome the students' difficulties, mistakes, and misconceptions by re-explaining the procedures of question completion which was not understood by the students.

Rosenkranzer *et al*. (2017) studied the effects of three different interventions namely technical course, didactic course and mixed course in student teachers' PCK for teaching systems thinking. Using a mixed methods approach, they collected data using a questionnaire with open-ended items and a quasi-experiment from 108 student teachers at the Universities of Education in Freiburg and Ludwigsburg in Germany. They used descriptive statistics and a coding scheme to show that student teachers' PCK for teaching systems thinking can be promoted in teacher education. In particular, they established that a technically oriented course without didactical aspects is less effective in fostering student teachers' PCK for teaching systems thinking.

Watson and Callingham (2013) considered the responses of 26 teachers from three Australian states to items exploring their PCK about the concept of average. The items explored teachers' knowledge of average, their planning of a unit on average, and their understanding of students as learners in devising remediation for two student responses to a problem. They used rubrics for assessing the responses and their results indicated a wide range of performances and a wide range of ability in relation to a hierarchical statistical PCK scale.

All the seven reviewed studies have been on measuring how much PCK teachers possessed (Ijeh, 2013; Watson & Callingham, 2013), the development of teachers' PCK (Rosenkranzer *et al*., 2017), how teachers applied their PCK (Ijeh & Onwu, 2012; Ma'rufi *et al*., 2018), instruments to measure PCK (Estrella *et al*., 2015) and how PCK influence students' achievement (Callingham *et al*., 2015). None of the studies has looked at the relationship of teachers' PCK and their teaching of statistics, a gap this study wishes to fill.

Method

Instruments

Self-Administered Questionnaire (SAQ). The SAQ had three constructs on PCK. PCK was conceptualized as the teachers' levels of general knowledge (6 items), pedagogical knowledge (6 items) and content knowledge (6 items). On the other hand, the instrument had 22 items on the teachers' teaching of statistics. The 40 items were scaled on a five-point Likert scale ranging from 1 (Strongly disagree) to 5 (Strongly agree). The reliability of the SAQ on multi-item variables was established through pilot testing.

Classroom Observation Guide. The classroom observation guide had three constructs to observe namely introduction skills (3 items), process skills (10 items) and communication skills (5 items) pertaining to teachers' general, pedagogical and content knowledge. Each of the 20 teachers selected was observed between 70-90 minutes, depending on the duration of the lesson. The number of teachers who possessed each of the 18 skills was counted.

Interview Guide. The interview guide had eight questions to probe into the teachers' PCK and their teaching of statistics. The interviews lasted between 45-60 minutes with each of the 10 teachers that were selected.

Document Analysis Guide. The document analysis guide had three questions that were used to analyse a teacher's scheme of work, lesson plan and lesson notes that the teacher carried to class. In particular, the study was interested in finding out the teachers' aims of teaching statistics presented to the students, the methods of teaching they indicate to use, examples they intend to use in class, and the teaching materials they prepare to use in class.

Sample

Schools. The sample consisted of ten secondary schools. In order to maintain confidentiality, the schools were given pseudonyms namely Aluhu, Boho, Chandru, Drula, Elonda, Fetha, Glenora, Hondalu, Ituli, and Jameyi. According to Kizza (2018), UNEB listed the best ten performing districts in Uganda's Advanced Certificate of Education (UACE) and in each of the district, it listed the best 20 schools. Wakiso was the biggest district of the listed districts and from its 20 listed schools, came the sample of the ten schools. These schools were both urban (6 schools) and rural (4 schools); and private (4 schools) and government (6 schools). The sampled schools had sufficient teaching materials (including computers and calculators), enough textbooks; enough teachers and comparatively similar performance in order to cater for extraneous variables.

Teachers. Out of each selected school, six statistics teachers were selected and thus, the participants in this study were 60 secondary mathematics [statistics] teachers. There were four times as many males (80%) as females (20%) who teach statistics in secondary schools. About 62 percent of the teachers had a teaching experience of more than 11 years and the majority (78.3%) had only a bachelors' degree. Meanwhile, the teachers who majored (70%) in mathematics at higher institutions are more than twice as many as those who offered (30%) mathematics as a minor subject. The majority of the mathematics majors had Economics as their second teaching subject. Almost 77 percent of the teachers were married. None of the teachers had a degree in statistics or a degree in teaching statistics but rather all of them had one or two course units in statistics at the University. Also, none of the teachers had received in-service training in teaching statistics.

Data Analysis

The average index of the content validity of the SAQ was found to be 0.78 which is an acceptable level (Amin, 2005). To achieve the three objectives of this study, the items of teachers' general knowledge, pedagogical knowledge, content knowledge and the teaching of statistics were described in terms of their means and their overall rating based on the five-point Likert scale. The relationships between general, pedagogical and content knowledge and the teaching of statistics were found using Pearson's Linear Correlation Coefficient. The data from lesson observations, interviews and document analyses were categorised and scored and also reported as narratives.

Findings

Teachers' General Knowledge and Teaching Statistics

The first objective of this study was to find the relationship between the teachers' levels of general knowledge and teaching statistics in secondary schools in Wakiso district. Tables 1 and 2 give the means and the ratings of the items of teachers' levels of general knowledge and the teaching of statistics respectively. These items were answered on a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). According to Tables 1 and 2, the teachers' general knowledge and teaching statistics had overall means of 3.36 and 3.09 respectively. These values indicate that teachers neither disagreed nor agreed to items rating their general knowledge and teaching statistics. Table 3 shows that no teacher carried other information sources to the classroom apart from textbooks and that a few (25%) teachers gave instructions in their lessons. Still, a few (35%) teachers introduced the lessons according to their lesson plans. Table 4 reveals that majority (65.0%) of the teachers whose documents were analysed carried their own lesson note books to class and a few (35%) teachers conducted their lessons following their lesson plans. Only 10 percent of the teachers had schemes of work.

Item	Mean	Rating
I hold productive conversations about mathematical ideas with my students	2.56	Neither disagree nor agree
I evaluate students' ideas	2.85	Neither disagree nor agree
I interpret students' explanations to the whole class while I am teaching	2.97	Neither disagree nor agree
I know and use definitions of terms and use them correctly	3.73	Agree
I use clear explanations about concepts	4.00	Agree
I consider key principles and ideas that underlie statistics while teaching	4.02	Agree
Overall	3.36	Neither disagree nor agree

Table 1: Teachers' General Knowledge of Teaching Statistics

Item	Mean	Rating
I read and explain the text book	4.23	Agree
Statistics does not need practical work	3.80	Agree
I need to cover as many problems as possible in past examinations	4.18	Agree
I dictate notes to the class since students are mature enough	1.55	Disagree
I use the lecture method to teach statistics	4.00	Agree
I sometimes use question and answer approach while teaching statistics towards examinations	4.00	Agree
Sometimes I leave the students to research and collect their own data	2.62	Neither disagree nor agree
I prepare for the lesson before I go to class	4.00	Agree
I endeavour to return students' written assignments in time.	4.00	Agree
I explain all the difficult concepts in statistics	1.33	Strongly Disagree
I criticize students for their academic mistakes	2.57	Neither disagree nor agree
I accept my students' suggestions	2.48	Neither disagree nor agree
I take remedial classes for the weak students	1.43	Strongly Disagree
I act as a guide to students' problems	2.53	Neither disagree nor agree
I make sure that I give clear instructions to students during practical and class work	4.00	Agree
I encourage myself to be positive to my students during statistics lessons	4.00	Agree
I reward for student academic performance	1.38	Strongly disagree
I try to communicate to students in a language	4.00	Agree

they understand		
I take interest in personal problems of my students	2.65	Neither disagree nor agree
I encourage my students during my lessons	3.73	Agree
I punish weak students for being lazy	2.48	Neither disagree nor agree
I have open conversations with my students	2.97	Neither disagree nor agree
Overall	3.09	Neither disagree nor agree

Table 2: Teachers' views on teaching statistics

Introduction Skills	Percentage scores (Number of Total Scores)
Lesson plan	35.0 (07)
Instructions given	25.0 (05)
Information sources apart from textbooks	00.0 (00)

Table 3: Teachers' Scores during Classroom Observations in Introduction Skills

Documents	Percentage scores (Number of Participants with them)
Scheme of Work	10.0 (02)
Lesson plan	35.0 (07)
Lesson Notes	65.0 (13)

Table 4: Teachers' Scores on Document Analysis

Classroom observations revealed that the aims of teaching particular topics presented in the teachers' documents centred mostly on the students being able to solve as many problems as possible and applying routine statistical formulae. Interviews revealed that the teachers had to rush through the syllabus in order to complete it before the final examinations. The rush left them with no opportunity to explain the nature of statistics and the aims of teaching it. They rather insisted on formulae and calculations such that students could be able to pass their final examinations. Concerning the nature of teaching statistics, the teachers indicated that teaching statistics basically emphasized definitions and calculations. Pearson's Correlation Coefficient, $r = 0.130$ between the scores on teachers' levels of general knowledge and teaching statistics and a p-value of 0.320 indicated that at the 0.05 significance level, the relationship between the two variables was not statistically significant, thus, there was no relationship between the teachers' levels of general knowledge and teaching statistics.

Teachers' Pedagogical Knowledge and Teaching Statistics

The second objective of this study was to find the relationship between the teachers' levels of pedagogical knowledge and teaching statistics in secondary schools in Wakiso district. Tables 5, 6a and 6b give the means and the ratings of the items of teachers' levels of pedagogical knowledge and classroom observations respectively. The items in Table 5 were answered on a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). According to Table 5, the overall mean of teachers' pedagogical knowledge (PK) was 3.31 indicating that teachers neither disagreed nor agreed to the items rating their PK. Table 6a shows that while all teachers did not use displays during teaching statistics, 90 percent of them used lecture methods. While 50 percent of the teachers used many examples on a particular concept while teaching and 45 percent of them used a meaningful statistic language to the students, only five percent of the teachers modified problems to make them either harder or easier to challenge students. Table 6b shows that only a few (5%) teachers listened to, as well as interacted with the students and all of them gave notes to the students. Because only 10 percent of the teachers gave time to the students, very few (15%) students were able to ask questions during the lessons.

Item	Mean	Rating
I modify problems to be easier or harder to test the students understanding	2.67	Neither disagree nor agree
I use a mathematical language which is easier in a variety of ways for my students and yet mathematically correct	3.37	Neither disagree nor agree
I recognize different ways to solve the same problem	3.47	Agree
I ask my students to make mathematical justification, conjectures and look for patterns of their answers and reasoning	2.40	Disagree
I chose useful examples while teaching	4.07	Agree
I select appropriate representations while teaching	3.90	Agree
Overall	3.31	Neither disagree nor agree

Table 5: Teachers' Pedagogical Knowledge of Teaching Statistics

Process Skills	Percentage Scores (Number of Scores)
Displays (e.g. charts, data) involved / representations	00.0 (00)
Examples used	50.0 (10)
Language used. Is it meaningful to students?	45.0 (09)
Modification of problems	05.0 (01)
Teaching methods used, lecture or inquiry?	90.0 (18)

Table 6a: Teachers' Scores during Classroom Observations in Process Skills

Communication Skills	Percentage Scores (Number of Total Scores)
Taking of notes	100.0 (20)
How often the teacher listens to students	05.0 (01)
The time the teacher gives to students to ask questions	10.0 (20)
Interaction between teacher and students	05.0 (01)
Ability of students to ask questions	15.0 (03)

Table 6b: Teachers' Scores during Classroom Observations in Communication Skills

Classroom observations showed that teaching was teacher-controlled and basically theoretical. Too much emphasis is placed on the application of statistical techniques rather than on discussion of the results, examination of the data and on the inferences which should be drawn in the light of the context of the data used. Teachers solved many problems picked from textbooks and past paper examinations with slight description and passed on a few questions to the students moreover, textbooks as are available concentrate on theory rather than practice. The teachers emphasized the presentation of correct formulae and correct answers, drawing correct graphs and drawing simple conclusions. Meanwhile, statistics concepts were generally not related to real life experiences. Document analysis revealed some lessons were either far behind or far ahead of what the teachers had planned. Some lesson plans' dates, classes and/or streams and number of students did not tally with the actual lessons.

During actual teaching, discussions were rare. Interviews revealed that according to the teachers, discussions consume a lot of time yet they have to catch up with the wide syllabus and hence, engage with discussions towards examination periods. The teachers also indicated that they gauged their students' understanding of statistical concepts through the marks that they obtained from assignments, tests and examinations. However, they would not know if the given assignments were copied from fellow students or even, if the high scores from examinations were a result of cramming formulae. Almost all interview responses were centred on students' passing of examinations. Pearson's linear correlation coefficient, $r = 0.004$ between scores in levels of pedagogical knowledge (PK) and teaching statistics and a p-value of 0.978 indicated that at the 0.05 significance level, the relationship between the two variables was not statistically significant, thus, there was no relationship between teachers' PK and teaching statistics.

Teachers' Content Knowledge and Teaching Statistics

The third objective of this study was to find the relationship between the teachers' levels of content knowledge and teaching statistics in secondary schools in Wakiso district. Tables 7 and 8 give the means and the ratings of the items of teachers' levels of content knowledge and classroom observation respectively. The items in Table 7 were answered on a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree). According to Table 7, the overall mean of the teachers' content knowledge (CK) was 2.64 which indicated that teachers neither disagreed nor agreed to the items rating their CK. Table 8 shows that all teachers did not evaluate students' ideas and only 10 percent of them gave clear explanations to the students. While only 15 percent of the teachers gave clear definitions of statistical concepts and did not use them appropriately, 30 percent gave the sequence of events while they taught. Only 25 percent of the teachers gave details of the topic at hand.

Items	Mean	Rating
I explain goals and mathematical ideas to students	2.32	Neither disagree nor agree
I emphasize appropriate levels of accuracy	2.43	Neither disagree nor agree
I recognize wrong answers, spot the method used to get a wrong answer and recognize when a right answer is a result of faulty thinking	2.45	Neither disagree nor agree
I find out what my students did wrong when I give exercises	2.50	Neither disagree nor agree
I build correspondences between models and procedures	3.02	Neither disagree nor agree
I am aware and use a variety of interpretation of figures and data	3.10	Neither disagree nor agree
Overall	2.64	Neither disagree nor agree

Table 7: Teachers' Views on Content Knowledge of Teaching Statistics

Process Skills	Percentage scores (Number of Scores)
Definitions given and how they are used	15.0 (03)
Details of topic given	25.0 (05)

Sequence of events	30.0 (06)
Explanations given. Are they clear?	10.0 (02)
Evaluation of students' ideas	00.0 (00)

Table 8: Teachers' Scores in Classroom Observations in Process Skills

The teachers' documents did not reveal greater emphasis to the selection and proper use of data representation and analysis. An examination of explanations, examples and analogies that the teachers use to guide students' development of an understanding of statistics indicated that most teachers' actions largely stem from an understanding based on having been taught in particular ways. Interviews revealed that the teachers hardly tried data collection with themselves and the students as well. They only used data from textbooks. Most teachers lacked the knowledge of how statistical concepts and topics spiralled and what topics students found either easy or difficult. They also did not recognise the importance of helping students orient themselves to data, thus, a lack of demonstrating knowledge of students' likely levels of understanding. Pearson's Correlation Coefficient, $r = 0.084$ between the scores on the levels of content knowledge (CK) and teaching statistics and a p-value of 0.522 indicated that at the 0.05 significance level, the relationship between the two variables was not statistically significant, thus, there was no relationship between the teachers' CK and teaching statistics.

Discussion

The first objective of this study was to find the relationship between the teachers' general knowledge and teaching statistics. According to the overall mean (Table 1), teachers neither disagreed nor agreed to items rating their general knowledge of teaching statistics. This implied that teachers were not sure of their general knowledge of teaching statistics. The teachers' rating was not surprising as interviews, document analysis and classroom observations revealed that the teachers' aims of teaching particular topics that they presented were majorly centred on the students being able to solve as many problems as possible and applying statistical formulae. In essence, this provides a poor foundation for the students for developing analytical and statistical literacy skills.

They will not have the capacity to make sense of data in context. Pearson's linear correlation coefficient revealed that there was no relationship between the teachers' general knowledge and teaching statistics. This finding is in consonance with Widodo (2017) who having attempted to support teachers to conduct lessons that facilitated students' reasoning, found that teachers' general knowledge was lacking and needed intensive and continuous support to enhance reasoning among the students. Further, this finding could be lent to the teachers' inadequate planning for the statistics lessons, as very few of them had lesson plans. This was evident during classroom observation. There was no preparation for teaching and lessons development was generally flat as there was no introduction of the lessons but rather emphasis on formulae and computations, an indicator that the teachers do not understand what is expected of statistics.

The second objective of this study was to find the relationship between the teachers' pedagogical knowledge and the teaching of statistics. Table 5 shows that corresponding to the overall mean, teachers neither disagreed nor agreed to the items rating their pedagogical knowledge (PK) of teaching statistics. This means that the teachers still, were not confident about their PK. The lack of PK by teachers limits and affects the teaching of statistics. The teachers expressed a predominant use of teacher-controlled teaching, where students have little interruptions and choices in reasoning, organizing or displaying results. One possible explanation of this could be pressures from the UNEB examinations policy that require teachers to complete the syllabus before the students' final examinations. This makes teachers to rush through the syllabus without emphasis on understanding. Another explanation is that there are no pre-service training programmes to offer statistics teaching methods and no efforts have been put in towards in-service training for the teachers already in active teaching.

Pearson's linear correlation coefficient revealed that there was no relationship between the teachers' pedagogical knowledge and teaching statistics. This finding supports what Opolot-Okurut, *et al* (2008) found to the effect that statistics is taught by teachers who have pedagogical knowledge to teach mathematics but hardly had an opportunity to develop sound knowledge on the principles and concepts underlying good practices of teaching statistics. This means that teachers are unable to deal with learners' conceptions in statistics. They do not know how to interpret students' explanations and cannot tell if the representations and examples they select are useful to the students because they are already laid down, step-by-step, in the textbooks and only need shallow explanations. Yet, according to Ijeh and Onwu (2013), mathematics teachers deal with learners' conceptions in statistics by applying topic-specific instructional skills and strategies.

The third objective of this study was to find the relationship between the teachers' content knowledge and teaching statistics. The overall mean according to Table 5 revealed that teachers neither disagreed nor agreed to items rating their content knowledge (CK) of teaching statistics. This meant that teachers were not sure of the statistics content yet, high quality teaching requires that teachers have a deep knowledge of content. Although in any teaching and learning process, teachers must possess considerable knowledge on the content to be taught, design appropriate approaches and plan for effective activities, this was not the case in this study. Document analysis revealed that key concepts were not articulated but instead there was a smattering of statistical terms and formulae. An examination of the clarity and explicitness with which the concepts to be taught were expressed in the lesson plans, the kinds of activities planned, and the teachers' lack of acknowledgement of student understanding all revealed the teachers' lack of content knowledge.

Pearson's linear correlation coefficient revealed that there was no relationship between the teachers' content knowledge and teaching statistics. This finding is in agreement with Callingham, Carmichael and Watson (2015). Despite their development of valid instruments to measure both student and teacher content knowledge and teachers' PCK, they found linking teachers' knowledge directly to students' learning outcomes elusive yet according to them, students' outcomes are influenced positively by their teacher's PCK. In a nutshell, the teachers' understanding of the content was not found to be good enough and therefore there was no quality teaching that was reflected during actual teaching. This could be explained by the fact that the content to be covered by the teachers is laid down by

UNEB without specific details but rather outlines. As a result, the teachers only cover, from the recommended textbooks and the outlined topics without the emphasis on details of the topics. Further, while the curricula provides an overview of what students should be taught, they do not address the issue of what knowledge is needed by the teachers of these students.

Conclusion

The study attempted to establish the relationship between teachers' pedagogical content knowledge and the teaching of statistics in secondary schools in Wakiso district. Using Pearson's linear correlation coefficient, it was established that there was no relationship between teachers' PCK and the teaching of statistics. This means that teachers' lack of pedagogical limits and affects the teaching of statistics in most schools. Also, teachers' lack of planning could be a reason. Unless teachers address these issues and are monitored, the outcry of poor performance shall remain an anthem in the country. The study however, had limitations. For example, the study was applied to only Wakiso district, which is one out of 119 districts in the country. The sample size could have been bigger as well. The teaching of statistics could also have other explanatory variables than PCK suggested by other frameworks. The sample of the study was small and not selected to be representative of the large population of secondary school teachers in the country. It was recommended that the Ministry of Education and Sports, National Curriculum Development Centre and Uganda National Examinations Board should identify the training necessary to equip teachers with pedagogical content knowledge to teach statistics effectively, revising the curriculum and emphasizing a practical approach in teaching statistics in teacher preparation programmes, designing modules for pre-service and in-service mathematics teachers in order to improve their PCK in teaching statistics and also emphasize practical use of real life data by providing guidelines and manuals to all secondary schools.

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Teachers' knowledge about the nature of mathematics: a case from Pakistan

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This study reports on one of the research components of the larger study i.e., describes mathematics teachers' knowledge about the nature of mathematics and the sources that shape their viewpoints rooted in their socio-cultural, personal and professional experiences. Twenty one teachers participated in the focus group discussions. The findings indicate that the mathematics teachers hold dualist views about the nature of mathematics. On the one hand, teachers believed that mathematical knowledge is divine therefore is irrefutable. On the other hand, they maintained that mathematical knowledge is a human creation, invented to facilitate observing religious practices and also to support human survival.

INTRODUCTION & THE CONTEXT OF THE STUDY

Understanding teachers' beliefs about mathematics has long been the research topic as research evidence shows that teacher's personal theories about mathematics, mathematics teaching and learning have a great influence on their teaching practice (Lofstrom & Pursiainen, 2015; Barkatsas, 2008; & Gates, 2006). Thompson (1984) and Ernest (1998) based on their wider experience of working in the field of mathematics education claim that any attempt in improving the quality of mathematics teaching and learning must begin with an understanding of the conceptions held by teachers. To address one of the key issues discussed in mathematics teacher education, this study explored the sources that shapes teachers' beliefs about the nature of mathematics.

This study is conducted in Karachi, which is a large metropolitan city in the province of Sindh in Pakistan. Karachi is one of the cities which encompass a large representation from different schooling systems. Twenty-one teachers from both Public and Private schools participated in the study. The focus group discussions were used as a method for data collection to understand the process that led to the development of teachers' views and perspectives of mathematics. The first step in analyzing the focus group data was to have the entire discussions transcribed. The verbatim transcripts along with the notes on nonverbal communications and gestures were read and reread to generate descriptive narrative.

MATHEMATICS: A DIVINELY CREATED BODY OF KNOWLEDGE

The teachers based on their religious faith viewed mathematical knowledge as a divine creation. While explaining their viewpoint they referred to mathematical patterns present in nature and the holy Quran which was revealed to Hazrat

Muhammed (Peace Be Upon Him) 1400 years back as frame of reference. The teachers perceived that Almighty Allah has created the universe in which the mathematical principles and patterns are part of the creation. The following are some of the quotes stated by the teachers:

1 Nasreen: Allah has created the universe using the mathematical structures but it is up to us how we make sense of it using our five senses

2 Sameer: (referred to Galileo to draw attention to the similar argument) and said, mathematics is the language with which God has created the universe.

3 Anam: The sun rises and sets at an appropriate time and this is due to the pre-planned setup of the universe which neither you nor me has created. This is Allah's creation which will go on and on irrespective of whether we observe it or not.

MATHEMATICS: AN INVENTED BODY OF KNOWLEDGE

The teachers acknowledged human contribution in developing mathematical concepts. While referring to the 'human contribution', the teachers specifically accredited mathematicians for discovering and inventing mathematical knowledge as they work intensively hard whereas others are considered as the users of mathematical knowledge. However, the discovery or invention process is considered to be a solitary process as teachers think that mathematicians work in isolation and produce new knowledge which is later used by all of us. The following are some of the quotes stated by the teachers:

1 Mohammad: In order to identify the direction of Qibla an instrument based on spherical trigonometry was invented and due to this invention locating the direction of Qibla from any part of the world was possible

2 Naseem: Islamic calendar was worked out to determine the proper day on which to celebrate Islamic holy days and festivals such as Shab-e-Qadr, Eid-Milad-un-Nabi, Jumat-ul-Wida, Eid-ul-Fitr, Eid-ul-Azha, etc.

Overall, the analysis showed that religious and socio-cultural experiences of teachers are the major sources that shape teachers' thinking about the nature of mathematics in the Pakistani context.

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Form one students' understanding of the inclusion relationships among quadrilaterals: A case of a school in Tanzania

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Abstract

Before embarking on an exploratory study of form one students' learning of hierarchical classification of quadrilaterals, a pre-test was administered to the students to find out their prior understanding of the inclusion relationship of quadrilaterals. The results were then to be used for designing activities that would ultimately assist students to hierarchically classify quadrilaterals. Therefore, although the pre-test covered many other concepts related to quadrilaterals and also other concepts in geometry, there were thirteen items that focused on the inclusion of some quadrilaterals into others. Analysis of the data was, however, confined to only eight items that were considered to be at the level of the students, as they all dealt with concepts that were supposed to have been covered in primary school mathematics. Thirty-six students in all took the test in an untimed environment and it was found that the majority of the students had a very limited knowledge of the inclusion relationship among the elementary quadrilaterals that formed part of the primary school syllabus. For instance, only about 25% of the students who took the pre-test were able to say, correctly, that all squares were rectangles but surprisingly more than 30% of them said, incorrectly, that all rectangles were squares. Similarly, slightly more than 41% said, incorrectly, that all parallelograms were rectangles but only about 16% were able to say the correct inclusion of rectangles into parallelograms. These results seem to suggest that students may not have mastered the inclusion relationship of quadrilaterals by the time they complete the primary school cycle. It is therefore recommended that the inclusion relationship of quadrilaterals be explicitly introduced to primary school geometry instead of having the partition classification of quadrilaterals pre-dominating the teaching and learning of the primary school geometry syllabus.

Key terms: quadrilaterals, hierarchical classification, inclusion relationship, primary school geometry

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focused exclusively on the inclusion of some quadrilaterals into others. Analysis of the data was, however, confined to only eight of the thirteen items that were considered to be at the level of the students, as they all dealt with concepts that were supposed to have been covered in primary school mathematics. Thirty-six students in all took the test in an untimed environment and it was found that the majority of the students had a very limited knowledge of the inclusion relationship among the elementary quadrilaterals that formed part of the primary school syllabus. For instance, only about 25% of the students who took the pre- intervention test were able to say, correctly, that all squares were rectangles but surprisingly more than 30% of them said, incorrectly, that all rectangles were squares. Similarly, slightly more than 41% said, incorrectly, that all parallelograms were rectangles but only about 16% were able to say the correct inclusion of rectangles into parallelograms. These results seem to suggest that students may not have mastered the inclusion relationship of quadrilaterals by the time they complete the primary school cycle. It is therefore recommended that the inclusion relationship of quadrilaterals be explicitly introduced to primary school geometry instead of having the partition classification of quadrilaterals pre-dominating the teaching and learning of primary school geometry syllabus.

Key terms: quadrilaterals, hierarchical classification, inclusion relationship, and primary school geometry

Student's strategies in mathematics word problem solving

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Word problems are among the components of school curriculum and are taught at all levels of education. Some tasks involving word problems have proved to be more challenging for some students than others. The aim of the study was to explore the strategies the fifth grade students use to solve addition and subtraction word problems, the reason why some word problems are difficult than others and the effects of using first and second language in mathematics word problem solving.

The sample comprised of thirty-nine students drawn from one school in Livingstone, Zambia. These students took part in a test of 10 word problems. Thereafter, five of them were randomly selected and interviewed. The study employed qualitative research design. Information was derived using students marked answer sheets and interview guides.

The study found that many students interviewed had difficulties to read certain mathematics text. As they read, the students committed errors such as mispronunciation and repeating of words. It was also found that subtraction word problems were more difficult than addition word problems for students to solve. Strategies used by many students in problem solving were similar though errors were noticed in some cases. The study further revealed that students use local language during small group discussions, and switch to English during a class discussion and when instructed by the teacher.

Based on these findings, the study recommended that apart from having teachers and students' mathematics textbooks, there should be also parents' mathematics textbooks. Students' textbooks should include a number of word problems from real life situation which could enhance critical or advanced thinking in them. It is also recommended that students should be allowed to discover and reinvent strategies by themselves than depending on available strategies reflected in mathematics textbooks or on those given by teachers.

INTRODUCTION

In the modern society, there are expectations on schools to ensure that all students have the opportunity to become mathematically literate. In this case, students are considered being literate in that they have an ability to set up problems with an appropriate operation and be able to use a variety of techniques to approach and work on problems, Siegler (1991) argues that children create their own symbolic tools to solve problems as well as using ones given to them. For example 7 to 11 years old have the ability to solve problems using informal knowledge. Informal knowledge refers to untaught methods used to solve a variety of arithmetic problem.

The questions of how children process their thinking in solving problems have been investigated over the years in the mathematics education community. For example, research has shown that children initially solve addition and subtraction word problems by directly representing the action or relationships in the problem (Carpenter & Moser, 1984; Hiebert, 1982; Franke & Carey, 1997). In this case, children`s failure to interpret questions is dependent much on the language employed. Some studies have shown that children who are capable of solvng arithmetic problems numerically encounter difficulties in problem solving when the same problems are presented in words (Bernardo 1999).

Bernardo (1999) states that children`s difficulties in problem solving is dependent on the ability to understand the mathematical problem structure that is within the problem text. In his research, Bernardo showed that "children`s failure to solve the problem come from an error in just one of the range of concepts and procedures applied" (P 149). The research conducted among Philipino children showed that children faced challenges in comprehending certain types of texts in word problems (Bernardo 1999).

In the document (educating our future), the Ministry of Education (1996) embarked on development of basic numeracy and problem skills as the priority target for primary mathematics education. The aim was to ensure that "those who leave school are able to function effectively in society, while those who continue in school have an adequate basis for further education" (p. 14). Later, in the Basic School Curriculum Framework (2000a), the Ministry of Education settled for the term essential numeracy, and identified its components. That is, on completion of primary school, a child would understand the meaning of the numbers from zero to one million, thus use the range of numbers to perform the four (4) fundamental arithmetic operations of addition, subtraction, multiplication and division. Since the 2 publication of Educating Our Future document in 1996, very little has been done practically to improve teaching and learning of numeracy problem solving in primary schools (Sampa 2004, Linehan 2005).

I wondered whether there was a connection between primary school children's unsatisfactory performance in numeracy and the dominant teaching methods used at primary level of education. Kelly (1991) argues that the prevailing teaching approaches at all levels of education in Zambia are inflexible and unimaginative, emphasising "factual knowledge and memorization" (p. 33). It is believed that teaching approaches were responsible for the difficulties many school children in Zambia are experiencing in learning and solving of mathematical problems (Kelly, 1991).

Many students at primary and junior secondary level use approaches provided to them by teachers or those reflected in mathematics textbooks to solve problems given. As a teacher and having taught students at junior secondary, I observed that many students use vertical method when given problems on addition and subtraction. In addition, students use tallying, concrete objects and finger counting to solve problems given. When using finger counting, I observed that students count from smaller to larger numbers. For example, to solve 9 plus 5, a student starts counting from 10, 11, 12, 13, 14, 15 and the last finger to be raised is considered as the answer.

I also observed that students' strategies become abstract as they advance in education.

Language switching was not an exceptional, as students switched between local language and English during group and class discussions.

From these considerations and my experience as a teacher, I developed interest to investigate strategies that fifth graders use in word problem solving, why some word problems are difficult than others and the effect of using local and English in word problem solving. The results of my findings could provide a unifying framework for the development of teachers' knowledge of mathematics and enhance the production as well as improving already existing mathematics resources.

Purpose of the study

Over the years, there has being a growing number of concerns involving students using rote memory skills despite having the ability to invent strategies to solve mathematical problems. In Zambia, very little is known of students inventing and using strategies in problem solving. Little research has been conducted to ascertain the strategies students use in solving various problems in mathematics and science. It is against this background that this study was conducted to find out whether students could invent strategies and use them to solve word problems, and the challenges they encountered in comprehending certain type of texts in word problems. It is imperative that student's experiences in problem solving are documented to enable stakeholders such as the ministry of education improve 3 existing or produce news one to be used by the Curriculum Development Centre (CDC) for assessment and evaluation purpose.

Research questions

This study sought to answer the following research questions;

- What are the effects of using students' first and second language in mathematics word problems solving?
- Why are some word problems difficult for children to solve than others?
- What strategies are used by fifth graders to solve addition and subtraction word problems?

Statement of the problem

Word problems are among the components of primary school curriculum. Research shows that word problems are relatively difficulty for many children at all levels of education, and that children must learn addition and subtraction operations before solving simple word problems (Carpenter, Corbitt, Kepner, Lindquist and Reys, 1980).

Solving word problems could be seen as a process of translating words into mathematics expression and then solving the problem. The study by Bebout (1990) showed that children could learn to represent word problems numerically.

Significance of the study

This study is significant in that it will provide valuable information to interested parties such as the government and agencies such as non-governmental organization (NGO) who would like to assist by providing necessary materials such as mathematical textbooks which would enhance the use of different strategies to solve mathematics problem.

This study will also provide information to scholars who may be interested in carrying further studies related to this area.

Transcript

For presentation purpose, the transcript below was based on student one (S1) and two (S2). Questions answered in this case are 1 and 2. For more information refer; https://brage.bibsys.no, phiri-2014

The dialogue below is based on the interview with student 1 (S1).

Q1. Everlyn has 144 spaces in her photo album. So far, she has placed 89 photos in the album.

How many more photos can she put in before the album is full?

1. Me: I want you to begin reading question 1 on your question paper.
2. S1: “Am not good at reading, but I will try to read the question.”
3. Me: Write the words space and album.
4. S1: “spice and albom”
5. Me: Good, what did the first part of the question mean to you? You can explain in

Chinyanja!

6. S1: [she remained quiet for some time until question in utterance 5 was asked]
7. Me: Look at the two words "photo album", what do the two words mean?
8. S1: "a photo album ilikwati buku yoyikamo ma photos" [it's like a book were photo are put].
9. Me: Ok, what does "144 spaces" in the 1st phrase of the question mean?
10. S1: "144 spaces mean that there are 144 ma places yama snaps" [144 places for snaps].
11. Me: How did you find the remaining spaces?
12. S1: "by subtracting."
13. Me: Why did you have to subtract?
14. S1: "the question required me to find the remaining spaces from 144"
15. Me: Explain how you found the remaining spaces.
16. S1: "Firstly I wrote 144 minus 9 equals to question mark. Then I borrowed 1 ten and then added it to 4 take away 9 equals to 5. So 100 plus 30 minus 80 equals 50, then 50 plus 5 equals 55."
17. Me: Why did you use this approach?
18. S1: "that's how my teachers taught me in my previous grades; I also copy from maths textbook.

The dialogue below is based on student 2 (S2) interviewed.

The student (S2) employed two methods in solving question 2. The first method is the use of drawings while the second one is where the student used the number line. The student explained the procedures leading to the attainment of the solution, though had difficulties in explaining how the number line was used in the test.

Q2. Mary has 3 packages of gums. There are 6 pieces of gum in each package. How many pieces of

gum does Mary have altogether?

19. S2: Reads question 2
20. I: How did you understand the question?
21. S2: “there are 3 packages and each has 6 pieces of gums.”
22. I: What were you required to find?
23. S2: “the total number of pieces of gums in all the three packages.”
24. I: How did you find the total number of gums?
25. S2: “I drew circles in rows and columns, 3 rows and 6 columns. Total count of circles gave 18 as the answer.” [Pointing to her drawing]
26. I: Ok, which other way was used to find the answer?
27. S2: “added 6 plus 6 plus 6 which is equal to 18.”
28. I: Good, why did you add 6, 3 times?
29. S2: “because there are 6 pieces of gums in each package and there are 3 packages.”
30. I: Alright, which other method did you use to calculate the answer?
31. S2: “I used a number line” [Pointing at the drawing]
32. I: How did you use the number line to solve the problem?
33. S2: “I firstly drew a straight line and marked from 0 to 40 at intervals of 5 units. First, second and third lines drawn above the markers are 6 units each in length. I counted all the intervals from 0 to 18; the answer I got was 18.”

Discussion of the results

Word problems are among the components of school curriculum. Most curricular programs show that word problems are relatively difficult for many students at all levels of education. Dwelling on the analysis of my empirical data, I argue that many students were successful in solving addition than subtraction word problems. Students come in contact with addition before getting admitted into school. Similar finding have been reported that subtraction word problems are more difficulty than additional word problems for students to solve (Bebout, 1993; Carpenter et al., Fuson et al., 1997). Furthermore, I argue that students' failure to solve some mathematics word problems is due to the language employed in the question and this argument is in line to that of Bernardo, (1999).

The findings also revealed that students solve word problems by representing the action in the problem. For instance, some children in this study drew circles to represent pieces of gums in a package before solving the problem. My findings coincide with other researchers who indicated that students initial solve word problems by directly representing the action or relationships in the problem (Carpenter & Moser, 1984; Hiebert, 1982; Franke & Carey, 1997). In the same vein, Lopez and Veloo (1993, 1994) argue that students at primary school perform better when asked to draw diagrams before solving the problem.

The results from the test in this study suggest that most students did better on tasks which required addition than those of subtraction. Therefore, I argue that some students add the values on the tasks which require subtraction and hence obtaining incorrect solutions. This argument is similar to that of Barwell et al, (2011) in which they have argued that students combine numbers in a problem without understanding the question and give unrealistic solution.

In this study, students were asked to read the questions and explain how they solved each problem. The findings revealed that many students encounter reading difficulties, for instance some words in the texts were pronounced wrongly; hence I argue that reading difficulties cut across students at all level of education and the society in general. Similar finding have been reported in which many students encounter reading and understanding difficulties in comprehending mathematical texts leading to solution errors (Cummins, Kintsch, Reusser & Weimer, 1988 cited in Bernardo, 1999).

In this study, I also observed that many students used mixed methods to solve word problems given. Similarly, Franke and Carey (1997) argued that students perceive mathematics as a problem solving in which different strategies are considered useful and hence use them to solve problems given. I argue that many students used vertical method and empty number line to solve the problems given. On vertical computation procedure, many students solved addition word problems by carrying and regrouping which also involved the formation of new group of tens. This procedure is similar to that of borrowing in subtraction and this argument resonates with findings by Varelas and Becker (1997).

Errors were also observed in some procedures, for example, a student subtracted 4 from 9 and wrote 5. Then, subtracted 4 from 8 in tens column and brought 1 down in hundreds column, and wrote 145 as the answer. In this situation, I argue that many students tend to subtract smaller numbers from larger numbers regardless of their position. This argument is similar to that of other researchers who argued that students' common errors when solving subtraction problems are either subtracting smaller from larger numbers or mistakes with borrowing (Dickson et al., 1984 cited in Resnick, 1982). I also observed that a student placed digit 8 under 47 instead of it being under 7 on unit column and still had the correct answer. Similar arguments have been presented by Dickson et al (1984) who argue that addition seems to present students with the least of the four operations though the common errors relate to the positioning of numbers in vertical presentation of addition and the process of carrying (Dickson et al., 1984 cited in Resnick, 1984).

Many students also used different counting strategies in problem solving. I observed that finger counting was used by many students who counted from smaller numbers. For instance, one student used pencils as counters to solve a given problem. The student grouped pencils in three sets and each set had six pencils. Then the student counted by putting together the pencils from the three sets and wrote 18 as the solution to the problem. Similarly, other researchers have reported that counting strategies could be carried on by the use of fingers, sticks, pencils, cubes or by counting mentally. For instance, grouping two sets together and using fingers or concrete objects to count all items, children start by counting the elements in the first set and finishing with those in the second one (Baroody and Standifer, 1993; Carpenter and Moser, 1983; Hugher, 1996; Nunes and Bryant, 1996, Groen and Parkman, 1972). Ma, (2011) argued that "counting counters" approach is mainly there to help students easily find solutions to addition and subtraction problems.

My findings also revealed that many students use counting strategies similar to those given to them by teachers, though there were few instances in which students strategies became abstract, for example, many students counted on from a smaller number by either using sticks, pencils or fingers. Similar findings have been reported by other researcher who argue that students use advanced counting strategies such as "counting on" or "counting back" to solve problems given (Carpenter and Moser, 1983; Riley et al., 1983). Therefore, I argue that as students grow and advance in education, they become more advanced in thinking and hence use strategies which are more abstract. In this case, many students move away from a situation where they count using concrete objects such as sticks, small stones, blocks and fingers. Similarly, Groen and Resnick (1977) argue that students develop advanced approaches to computation as they grow.

Language switching was observed during the interview as students switched between local (Chi Nyanja) and second language (English). Many students in this study used local language in expressing their ideas about a problem. I argue that students use first language when discussing in small groups or explaining certain concepts to their friends. English is mainly used in a whole discussion when instructed by the teacher. Similarly, Setati (2009) and Kazima (2009) argue that students switch languages when working on solution to task within their small groups and when it comes to class discussion, students contribute only when they are requested by the teacher and when this happens, they use second language. Clarkson (1994) also argued that children's use of local language as they solve mathematics problems performed well in a test, an indication that home language is advantageous in a school setting.

In line with the above arguments, the Ministry of Education (MoE) in Zambia reversed its education policy requiring the pre - school and early grades (1 – 4) to be taught in all subjects in regional languages and English be taught as a subject (MoE, 2012). Based on the analysis of my findings, I argue that the use of local language in teaching and learning would improve students' performance in mathematics and other subjects which are taught in English. Bernardo (1999) also argued that students whose tasks are written in local language perform better than those with tasks written in English. Similarly, Adetula (1990) and Bernardo (1999) argued that students involved on tasks written in local language improved their performance than before. Furthermore, Clarkson (1994) argued that students' use of local language could be advantageous in schooling setting. He further argues that students' competence in their local language does affect their achievement scores in mathematics positively.

Conclusion and Recommendation

Conclusion

From the finding of this research, it is clear that students were successful in solving addition and subtraction word problems. Students were more successful in solving addition word problems than subtraction word problems. This finding concede with the result from previous research that showed subtraction word problems as being more problematic than addition word problem for students to solve (e.g. Bebout, 1993; Carpenter et al., 1993; Fuson et al., 1997). This difficulty became more pronounced when borrowing was required.

In solving mathematics word problems, different strategies such as vertical, empty number line and counters were used. The results however, showed that the number line was wrongly used by all students despite using it to solve problem. The other research was where students count markers rather than the intervals. It is also clear that students used different counters such as pencils, small pieces of sticks and fingers to help them find solutions to problems given. Counting counters are commonly used by primary school going students though finger counting is applicable to most students at all levels of education.

It was a common observation that students switch languages during discussions within small groups and class discussion. First language is used when students help each other to understand certain mathematics concepts (Bernardo, 1999; Setati, 2009). Similar research conducted showed that students performed well during the test which was written in their local language (Kazima, 2009; Bernardo, 1999; Setati, 2009; Clarkson, 1994). My research is consistent with the results from previous research that shows home language as being advantageous than second language in a school setting.

Recommendation

Based on the findings of this research, the following are the recommendations;

1. The government through the Ministry of Education and other stakeholders to embark on producing mathematics handbooks which parents could use to help children with problem solving at home.
2. The Ministry of Education through the CDC should include mathematics word problems based on real life which would enhance critical or advanced thinking in children.
3. Children at all levels of education should be given opportunity to re-invent strategies on their own or under the guidance of teachers than depending on those strategies given in mathematics textbooks.
4. The Ministry of Education should also embark on publication of mathematics textbooks in local languages for all levels of education, as this would enhance easy understanding of mathematical concepts by children.
5. The Ministry of Education should encourage workshops where teachers and parents meet to discuss ways of how best to improve mathematics performance among children.
6. Refresher courses are necessities for teachers, as this would enable them acquire latest information based on mathematics education. This on the other hand would also improve their teaching skills.
7. The mathematics textbooks at primary level should include activities or games which could inculcate in children the concepts of addition, subtraction, multiplication, division and counting.

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Understanding the meaning of the equal sign

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Much research about students` understanding of the equal sign record that most students view the equal symbol as a signal to carry out a computation instead of a symbol expressing mathematical equivalence. The purpose of this study was to find out the meanings Zambian Grade 8 students assign to the equal sign and how their assigned meanings of the equal sign affect their performance in solving equations. A test and individual interviews were used to collect data and the assigned meanings were compared across students as they used the equal sign in solving equations with the aim of correlating between understanding the equal sign as a symbol representing equivalence and success in solving equations. It was found that students who demonstrated an ability to recognize a relational meaning for the equal sign scored higher in solving equations.

INTRODUCTION

Expertise in mathematics is seen as an essential tool to success in modern society as it is used in our daily life. In line with such thinking, mathematics has been considered as one of the most important subjects in the school curriculum. The students` under achievement in mathematics is a global concern (Pisa, 2003). This concern has led to research to finding out the reasons for students` low achievement in mathematics. Students` understanding of mathematical concepts in general have been problematic in most Zambian schools as observed in the mathematics pass percent at national junior secondary school leaving examination at Grade 9 level in most Basic schools in Zambia. In 2008, mathematics at middle basic level had a national mean performance of 39.8 % which is below pass mark of 40 % in the subject (Education, 2008).

Introduced by Recorde in the 16th century, the equal sign "=" symbol has become the universally recognized symbol to indicate mathematical equality (Cajori, 1928). Mathematical equality can be defined as the principle that two sides of an equation have the same value and are thus interchangeable (Kieran, 1981) It is among the symbols first met by students upon starting school and yet research show that many students have some misconceptions about the symbol even at secondary school level. A well-developed conception of the equal sign is characterized by relational understanding, realizing that the equal sign symbolizes the sameness of the expressions or quantities represented by each side of the equation (Baroody & Ginsburg, 1983). Students need to have this critical understanding of the equal sign in order to find logical conclusion when solving equations.

As a relational symbol, the equal sign gets different meanings dependent on the contexts in which it is used, For example, it can indicate an identity, define a function, at other times used with a placeholder and interpreted as a “do something” sign or as a sign that says “now follow the answer”. While in algebraic equations, the symbol is an example of an equivalence relation and can be used to designate symmetric and transitive character between the left- hand and right- hand side of the equation, for instance in $6x + 2 = 8 + 4x$ (Kieran, 1992).

It has been noted however, that relatively little attention has been given to finding out students` conception about the equal sign in developing countries like Zambia. It was found prudent to investigate the meanings Zambian Grade 8 students assign to the equal sign and how these relate to their performance in solving equations. This knowledge may help in creating a general overview of how students understand the equal sign in Zambia specifically and add to existing knowledge worldwide.

RESEARCH METHODOLOGY

The purpose of my study was to identify students` assigned meanings of the equal sign and how these meanings impacted on their performance in solving equations. As the purpose of my study is to provide a holistic, in-depth account of the case under study, extensive, multiple sources of data are needed (Erickson, 1986). Triangulation is the term used to indicate the use of multiple pieces of evidence to claim a result with confidence. This increases the credibility or trustworthiness of the findings (Johnson & Christensen, 2008). For example in my study, I used students` written work, interview transcripts, and researcher`s notes to triangulate the data and arrive at valid conclusions about students` understanding of the equal sign.

RESEARCH DESIGN

A qualitative research design was adopted for this study. Maxwell (2012) identifies five particular research purposes for which qualitative studies are especially suited. These are; to understand the meaning of the events, situations and actions involved to understand the particular context within which the participants act, to identify unanticipated phenomenon and to generate new grounded theories, to understand the process by which events and actions take place and to develop casual explanations.As this study explored students` assigned meanings of the equal sign and how the assigned meanings relate to performance in solving equations, these are questions concerned with the process of phenomenon which are best answered through qualitative paradigm(Creswell, 2012).

DATA ANALYSIS

After going through individual student`s answer scripts and think-aloud interview transcripts and coding the data it was observrd that 93.3% of participants gave an operational meaning of the equal sign compared to only 4.4% who gave a relational meaning. From the analysis of the written test, only 4% of the students got question 2(a) correct. 96% of the students put 42 in the box as the missing number in the mathematical sentence 14 x 3 = ☐ – 3. 100% of the students got question 2(c) wrong. The prominent answers were 4 and 9 for the mathematics sentence, 9 – 5 = ☐ - 9. 2% of the students got question 2(c) correct. 98% of the students settled for 3 as the answer. For question 2(d), no student got it right. The mathematical sentence 100 ÷ 5 = □ + 5 was treated the same as these other question, a and b where the left hand side had the operation and the answer comes just after the equal sign. All the students put 20 in the box.

In question 2(e), 2% of the students did not attempt it, 4% got it correct while the rest got it wrong. This question was most interesting because of the wrong answers, 2 answers were most prominent and these were 13 and 169. The mathematical sentence was $\frac{169}{13}$ = 13 - □. Student who wrote 13 in the box divided 13 into 169 and got 13 which was put into the box.

RESULTS OF THE STUDY

This study`s findings are consistent with previous results by Alibali et al. (2007) that suggest that students` misconceptions may be due to instruction and textbooks. When students are exposed to same procedure of solving equation, this maybe ingrained in them to think that it is the only way to do it. If instruction or textbook examples are not varied, students may believe that unless they follow the laid down procedure, they may not be doing the right thing. In a context where a procedural interpretation of an equation is consistent with a student`s perception of how they should interact with the equation, they will most likely fail to interpret the equal relation. From this study, it is interesting to note that Zambian grade 8 students hold this operator meaning as a necessity in order to be seen to be doing something even when they know that they are not doing the right thing. From my analysis of their understanding of the equal sign, I came to a conclusion that actually for these students, the operator notion was not a misconception but rather a favored notion over relational meaning when not sure of what to do. Students had to do something in order to move on regardless of whether it made sense or not. When solving equations, most of the students went straight to do some computation without looking at the whole sentence. Their focus was on finding the answer, giving no attention to the structure of the whole sentence.

IMPLICATIONS TO TEACHING

In order to get a student to a point of really understanding what a symbol like the equal sign is communicating, teachers need to provide meaningful contextual and coherent experiences to nurture and develop this progressive understanding The concept of equivalence with respect to the equal sign is a concept that students must have a chance to work with in variety of contexts. If students are not exposed to appropriate multiple forms and representations of the equal sign, they will never be able to interpret and use it correctly. The language a teacher uses to instruct can reinforce the notion of procedure versus understanding in mathematics for example using phrases like “do the same thing on both sides of the equation” without elaborating the reason why can be confusing. If mathematics is taught as a collection of isolated facts and skills that have to be mastered separately it may lead students to decide at some point that understanding was not necessary but that learning the procedure was more appropriate and hence start looking for answers instead of understanding. A very disturbing outcome that should be noted in students who take this stance is that they rarely look for reasonableness of results (Lindvall & Ibarra, 1980).

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Evaluating and refining adapted mathematical knowledge for teaching instruments using item response theory

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Application of Item Response Theory (IRT) continues to be common among researchers interested in educational achievements using paper-and-pencil tests. In this paper, we review some pertinent issues in IRT application and demonstrate how we applied IRT to evaluate and refine adapted Learning Mathematics for Teaching (LMT) instruments. Data was collected by administering the adapted instruments to 212 and 2,135 pre-service primary school teachers for pilot and evaluation studies respectively. A 2-PL IRT model was used to calibrate pilot items and the results were used to construct shorter versions of the instruments: 20-itemed Form A and 21-itemed Form B with 6 anchoring items. The shorter forms were used in the evaluation study. The items varied in their discrimination (slope) (0.18 to 1.33), and their difficulty (location) mirrored a substantial range of MKT (ability,θ) (–3.46 to 4.98). However, the items as a set were most discriminating at higher levels of MKT. In the validation study, IRT scores and percentage of correct correlated at .90. We argue that when used appropriately, IRT can be an effective tool for evaluating and refining instruments.

Situational analysis on the teaching and learning of statistics and probability in Rwandan Teacher Training Colleges (TTCs)

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Abstract

The importance of statistics and probability in our daily life has led to many countries to include statistics and probability in the early age of their educational system. In Rwanda, statistics and probability is taught as part of the mathematics syllabus in all educational level. This paper investigates the teaching and learning of statistics and probability in Teacher Training Colleges (TTCs) in Rwanda as they are prepared to teach this subject in primary education. The paper focuses on the content knowledge, the attitude, and the pedagogical skills that are appropriate to train primary school teachers. As the syllabus suggest the teaching based on real-word data, it is imperative that primary school teachers be equipped with appropriate content knowledge, pedagogical skills, and right attitude towards statistics and probability. It is hypothetically believed that when children develop positive attitudes towards a given subject at early age, they are likely to understand the subject in higher level. Findings of this paper are drawn from a large study of my PhD programme that consists in working with TTC teachers to investigate active teaching approaches using ICT for developing preservice teachers' competences to teach in primary education. The results were drawn from questionnaire administered to 26 mathematics teachers from 13 TTCs to analyze the situation on current practices. Participants demonstrated positive attitude and the mastery of subject content. However, teaching focuses on the computation of statistical parameters only. There is also absence of ICT integration in the teaching and learning process.

Key word: Pedagogy, content, attitude, statistics, probability

Mathematical knowledge for teaching addition, equivalence, division and proportionality of fractions

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Mathematics teachers continuously plan and coordinate tasks of teaching to allow for meaningful interactions between learners and mathematics subject matter. Consequently, mathematics teachers ought to have multifaceted knowledge for teaching mathematics, which takes into account subject matter, students, and the art of teaching. Examining mathematical knowledge for teaching (MKT) demands sundry assessments. In this study, the mathematical knowledge for teaching addition, equivalence, division and proportionality of fractions for twelve primary school serving teachers was investigated using a two-tier test. The first tier cover common content knowledge (CCK) while the second tier cover pedagogical content knowledge (PCK). The two-tier test used with the MKT as a frame to assess teachers' MKT provided an opportunity for a better clinical diagnosis of MKT than traditional one-tier MKT tests. The results show that the teachers had sufficient common content knowledge to solve the problems in the first tier. However, they lacked the associated pedagogical content knowledge. Implications for using two-tier tests in MKT assessments are discussed.

A commognitive analysis of grade 12 learners' participation in a situated mathematical activity

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This paper report on grade 12 learners' mathematical discourse practices during their participation and engagements in the mathematics tasks that appear far removed from school mathematics in a mentoring intervention program. Drawing on Sfard's (2008) commognitive notions of mathematics discourse, our analyses show that learner were dominated by word use followed by visual mediators, routines and partially endorsed narratives. We argue that getting learners involved in unfamiliar real-life activities affords them with new learning experiences.

INTRODUCTION

There are generally two kinds of activities that are used in mathematics classrooms: those perceived to have visible connections to mathematics and those that are far-removed from mathematics. In this paper we report on an analysis of learners' experiences of an activity of the second type: the Kangaroo and Crossing the River activities. Mathematical process skills such as communication, representing, connecting, reasoning and problem solving (NCTM, 2000) appear to be the mostly displayed skills in the contextualized challenging game tasks such as these.

Theoretical Grounding

Participation is fundamental to social learning. It consists of the activity; the act of doing something or taking part, and the belonging; established through making a connection with others taking part. In the context of this study, 'participation' refers to grades 12 learners' engagement in a practice to become competent and legitimate members in mathematics discourse. Hence a theory of thinking as communicating (Sfard, 2008) informed our analysis of the learners' dialogues as they engaged in the mathematics tasks made available to them while participating in a mentoring program. Sfard (2008) identifies special features of a mathematics discourse thus; *key words, visual mediators, routines* and *narratives*. In order to be legitimised as a full participant in mathematics discourse, one needs to display all four features. The research approach was qualitative, located within an interpretive and exploratory stance in that we attempted to look for patterns in the data collected from classroom mentoring observations, and written work on the mathematics tasks.

Data analysis and finding

The analysis showed that learner interactions were dominated by word use followed by visual mediators, routines and narratives. Learners made limited use of routines that resulted in endorsable narratives. We argue that although this was the case, the Kangaroo and Crossing the River activities appeared to have provided new experience of learning mathematical concepts linked to number patterns, algebra and functions among learners. Although the tasks used appear far-removed from mathematics, they are viewed as more appealing and appropriate for encouraging participation of all learners, and hence enable learners to engage in more desirable experiences that are critical for gaining access to mathematics. Hence we argue that getting learners involved in unfamiliar real-life activities affords them with new learning experiences which, if sustained, can "spark" more lasting and new ways of seeing mathematics.

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Sub-theme 6

Assessment And Evaluation Issues In Mathematics Education

Assessment for learning in Africa: insights from classrooms in Tanzania

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This paper reports on teachers' practice regarding Assessment for Learning (AfL) mathematics in a study where AfL pedagogy was used to improve the quality of teaching. It draws on a multi-country research project entitled 'Assessment for Learning in Africa' that aimed to generate knowledge about how to develop and sustain teacher capacity in integration and use of assessment for improving learning in mathematics in challenging educational settings such as those in Tanzania. While AfL has the potential to significantly impact on improving student learning outcomes, there is a policy blind spot in international development on teachers' assessment in particular in low-income countries. Results showed that teachers' had developed contextually relevant approaches to assessment for learning. However, certain structural barriers in the classroom environment hindered the potential of AfL in the classroom.

INTRODUCTION

Tanzania including mainland and Zanzibar islands is a country of more than 42 million. Rapid urbanization is a demographic trend in Tanzania and the increase in the urban population is much higher in proportion to the region's rural population. Currently, the proportion of the country's urban population grows at a rate of approximately 5% per year as compared to the national average growth rate of 2.7% (UNDP 2015). Dar es Salaam is a rapidly expanding city and, in spite of its higher HDI score, it has within it huge disparities with acute poverty in unplanned dwellings (UNDP 2015; Lugalla & Mbwambo 1999).

Formal Education in Tanzania constitutes two years of pre-primary education, seven years of primary education, four years of Junior Secondary (ordinary Level), two years of Senior Secondary (Advanced Level). The country has made strides in providing access to primary education primarily due to strong policy commitment to education since its independence in 1960, where successive governments have seen education as necessary for development. However, the quality of learning processes and outcomes is low (Uwezo 2011, p.7). In a study of the plight of young children and youth in cities in Tanzania, UNICEF (2012) maintains that Dar es Salaam has one of the highest proportions of children living in unplanned settlements in sub-Saharan Africa (UNICEF 2012 p.64).

METHODOLOGY

'Assessment for Learning in Africa' is a three-year (2016-2019) project being carried out in six purposively selected schools in an informal settlement in Dar es Salaam Tanzania. The selected schools were under-resourced and class sizes were large (average n>80). The study included quantitative data from baseline and end line tests of students' performance in a specially designed mathematics test administered to more than 500 students. Along side a teacher development program was offered to all the mathematics teachers in the six selected schools. It comprised of workshops to explore teachers' perspectives about AfL; introduction of selected strategies and approaches for AfL in challenging contexts; and engage teaches in reflection on issues arising for AfL. Lesson observation and post-observation meetings of teachers with their mentors focused on analysis and evaluation of the lesson to understand issues in implementing Afl in their classroom. This paper draws on qualitative data from the teacher development component as follows:

No	Activity	Number	Data generated
1.	Lessons Observed (grade 4)	48	Observation schedule, fieldnotes, artefacts
2.	Mentor's Visits	48	Mentors notes
3.	Workshops	08	Workshop plans and reports
4.	Teachers' reflection	48	Teachers' writing on lesson evaluation

Framework of analysis was mainly drawn from the works of Wiliam (2006) and Hopfenbeck (2015) as discussed below. Research team across Tanzania and UK developed a coding scheme that included the key principle of AfL.

LITERATURE

Assessment in education is typically seen with a focus on outcomes in high stakes testing. Assessment *of* learning from such an evaluator position typically occurs at the end of a teaching unit or at the end of an academic year and is summative in nature. On the other hand assessment *for* learning is formative in nature as it is essentially concerned with how assessment can take forward the process of learning. In their seminal work Black and Wiliam (1998) looked into the 'black box' of classroom to look at formative assessment in the course of teaching and learning in the classroom and maintained that assessment becomes formative in nature when, "evidence is actually used to adapt the teaching to meet the needs of the students"(p. 2). Wiliam (2006) proposes five key strategies that underpin good practice in assessment for learning:

Clarifying and understanding learning intentions and criteria for success

Engineering effective classroom discussions, questions and tasks that elicit evidence of learning
Providing feedback that moves learners forward
Activating students as instructional resources for each other, and
Activating students as owners of their own learning" (Wiliam, 2006)

Along similar lines but in the context of Norway, Hopfenbeck (2015) maintains that in the Norway Education Act the main purpose of assessment is for learning based on the following principles:

(1) Students should be able to understand what they are going to learn and what is expected of them.
(2) Students should get feedback that informs them about the quality of their work and their level of achievement.
(3) Students should be advised on how to improve their learning outcome.
(4) Students should be involved in their own learning process and in self-assessment. (Hopfenbeck, 2015, p.45).

A significant element of the above principles of assessment *for* learning is that the onus of learning is on the students and the teachers' role is to create a facilitative environment for students' learning.

RESULTS AND DISCUSSION

The project is ongoing and analysis is still at a very preliminary stage. However, some trends and patterns emerge in the results. Lessons observed had a three-phased delivery structure. In the first phase the teacher introduced the topic, shared the objectives of the lesson often making reference to the previous lesson. The main body of the lesson followed where the teacher explained a mathematical procedure or the concept. During this phase the textbook and the chalkboard were the main resource for teaching. In the third phase students worked in their notebooks at 'exercises' taken from the textbook. Teachers were seen to employ a range of strategies to elicit evidence of students' learning and to provide them feedback on their learning. What follows is a brief description of the main strategies used. For consistency all data excerpts are from School Six.

<u>Use of chalkboard</u>: The classrooms were crowded and a large chalkboard along the width of one wall was found in each classroom. Teachers used the chalkboard creatively for a variety of purposes. In all cases the chalkboard was divided in to three sections, with the main and sub-topic written in the left hand column. Teachers wrote on the chalkboard exercises taken from the textbook, as many students did not have the textbooks. They would demonstrate worked examples on the chalkboard. In case they assigned individual or group tasks to the students, they were invited to present their work on the chalkboard. To accommodate the demands of the large class size, two or three students would be invited simultaneously to present their work on the chalkboard divided into columns to let each students work be represented separately. They invited students to check whether or not their peer's 'answer was correct'.

Use of tasks: Mostly teachers set tasks that were closed ended with only one correct answer. However, sometimes they also set open-ended tasks. For example in the lesson on addition of money (Tanzanian Shillings and cents) she asked the students, "Provide a word problem that entails use of multiplication of money". One student gave the example, "If One class has 25 pupils, how many pupils are in 3 classes?" The teacher wrote this on the chalkboard, applauded the student for giving the word problem, and went on to clarify that the particular example did not involve multiplication of money. She then invited another student to provide such a word problem.

Use of questioning: Teachers often used questions to help students move forward with mathematical procedures. For example a number of multiplication tasks involving carry-over with only one digit for the multiplicand.

312 144

x 5 x4

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1560 576

Students were invited in turn to the chalkboard to present their work and the teacher asked questions in order to make explicit the process of thinking when multiplying.

T: 2 times5 equals 10, how much do we take in head? Students chorus: 1

The above process of question and answer went on until the multiplication was complete. However, if a student made a mistake such as providing wrong multiplication facts (e.g. 4 x 4 = 12) she corrected the mistake but gave a general kind of feedback "she made a mistake because she does not know the tables" (*amekosea kwa sababu hajui tebo*).

Group work: Use of group work was observed in all the classes, partly because group work could be a useful strategy to promote students' discussion

Due to large number of pupils, noise and other distractions were generated, since the classroom was small and did not allow many movements for pupils to attend to the activities given. As a result, the teacher spent more time in trying to stop the distractions, but the pupils did not stop until the teacher went outside and came back with a stick, she threatened to beat them. Seeing the stick, most of the students stopped making noise, however, few continued, not until the teacher called them by name.

The illustrative examples above show that teachers used a variety of strategies to elicit evidence of students' learning. However, the extent to which the information received was used to provide feedback to the students' about their learning remained a question. For instance, in the case of multiplication of money, multiplication with carry over (e.g. 'holding in your head'), not knowing the tables for multiplication facts, it was not apparent if students' understood the place-value of digits. Teachers identified what was the mistake or the wrong answer but there was little evidence of probing why the students had provided the wrong answer.

Concluding remarks

To conclude, teachers employed different strategies to seek evidence of students' learning within the constraints of large class size and limited resources. However, teachers' creativity was constrained by a limited use of information drawn from interactions with the students. Moreover, issues of discipline and management of a large number of children in a confined space raised several challenges for them. The paper illustrates well the tensions in ensuring access and quality of students' learning in mathematics. It raises questions for policy and practice in improvement of mathematics teaching and learning in Tanzania and other low-income countries.

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Textbooks' errors and students' misconceptions: A case of one secondary school in Tanzania

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The paper aims at sharing the early results of an ongoing action research at one secondary school in Kibaha, Tanzania. The problem is how do mathematical errors in textbooks contribute to students' misconceptions? The school in question is a private secondary school with 74 students in forms I to III, but the investigation is concentrating on the Form III progress. The study, which is still continuing involves three mathematics textbooks used by students in forms I, II and III and students' responses to problems; oral and written. Three errors/ambiguities: solving inequalities, similar triangles and determining the range of a function, one from each textbook are analysed. Results so far show that mistakes and/or unexplained solutions in their textbooks very likely have led to students' misconceptions about these concepts. This is evident from their written solutions and oral justifications of their solutions. The paper discusses some of their written solutions along side corresponding mistakes from the respective textbook. Analysis for frequency indicates that errors/ambiguities in the determination of range of a function had a greater negative impact.

INTRODUCTION

Textbooks are one of the key elements in ensuring the delivery of quality education (UNESCO, 2015). In Tanzania, like in many other countries, the majority of secondary schools rely on textbooks as the main source of formal education. Shortage of qualified mathematics teachers in schools, especially in ward schools increases the dependence on mathematics textbooks for secondary school students in Tanzania.

Oxford University Press published the three "Mathematics For Secondary Schools" textbooks that are being analysed for errors, in this paper. They are Form One, Form Two and Form Three student books (Said, M., Mwambui, R & Owondo, V. (2009)). They have been in use since 2009. In all three textbooks, the errors and misconceptions that are discussed here emerge from written examples. Three examples are discussed, one from each textbook and are referred to as case 1, case 2 and case 3, respectively from Form One, Form Two and Form Three textbooks.

The group of students who are under this study are the current Form III students. They are 20 students who have used all the three textbooks since 2016 when they were in Form I. At the time, they were 15 in total. The students use the textbooks mainly for self-study and homework, whereas their teacher uses these books and other teaching (and learning) materials to prepare and teach.

The study used document analysis and discussion to gather data. Homework was checked, tests set and marked, and class discussion conducted during the investigation.

ANALYSIS AND DISCUSSION

Case 1: Form 1 textbook, page 140, Example 8.17, part (c).

Example 8.17

Solve the following inequalities:

(a) $\frac{^-1}{5}x < 2$ (b) $^-7x \geq 21$ (c) $3x + 6 > 2x + 12 > 20$

Solutions

(a) $\frac{^-1}{5}x < 2$

$^-5 \times \frac{^-1}{5}x > {^-5} \times 2$

$x > {^-10}$

(b) $^-7x \geq 21$

$\frac{^-7}{^-7}x \leq \frac{21}{^-7}$

$x \leq {^-3}$

(c) $3x + 6 > 2x + 12 > 20$

$3x + 6 > 2x + 12$

$3x - 2x > 12 - 6$

$x > 6$

Or $2x + 12 > 20$

$2x > 20 - 12$

$2x > 8$

$x > 4$

$x > 4$ or $x > 6$

$\therefore$ $x: x > 6$

Although the final answer given is correct, $x > 6$, the method of solution leading to this answer is incorrect. The misconception that may be picked by a learner, from the solution in the textbook are twofold: the incorrect use of `and' and `or' when solving inequalities, and getting the correct solution set when the inequalities are connect by `or'. Choosing to use `or' (instead of `and') means elements from either set will satisfy the inequalities. This is not correct when solving a compound inequality. The authors went on and made another error by picking the wrong set which satisfies $x > 6$ **or** $x > 4$. The correct choice from `$x > 6$ **or** $x > 4$ ' is $x > 4$ and **not** $x > 6$. The set $x > 4$ does not solve the given compound inequality because there exists at least one element in the set that does not. This suggests the reason why the authors opted for $x > 6$.

The problem involves solving a compound inequality. To do this, one needs to obtain a set of real numbers x that satisfies both inequalities:
$3x + 6 > 2x + 12$ **and** $2x + 12 > 20$. This then implies that $x > 6$ **and** $x > 4$, meaning that the required solution is the set of all real numbers greater than 6, the intersection of the two sets.

The fact that authors went on and solved another compound inequality (of different type - Example 8.19) correctly, made students believe that there are no conditions on using "and" and "or". This was revealed during class discussions. This misconception may have been passed to students through their teachers since attempts in their homework indicates so. Below is one of those attempts.

Soln $\lvert x+3 \rvert < 6$ $\pm(x+3) < 6$ Either $+(x+3) < 6$ or $-(x+3) < 6$ $+(x+3) < 6$ $\quad$ $-(x+3) < 6$ $x+3 < 6$ $\quad$ $-x-3 < 6$ $x < 6-3$ $\quad$ $-x < 6+3$ $x < 3$ $\quad$ $-x > 9$ $x > -9$ $\therefore x < 3$ or $x > -9$	This is one of the ten (out of fifteen) similar solutions. Surprisingly, they are all marked correct. It is not difficult to check that the answer is not correct. Take for example $x = 4$, one elements of the solution set because it is greater than -9. Substituting this value into the inequality, we have $\lvert 4+3 \rvert \nless 6$. No student, in this case, was able to explain why they wrote $\pm(x+3) < 6$. This shows that students do not understand what they are doing and hence cannot spot errors (in textbooks) themselves.

Case 2: Form 2 textbook, page 89, Example 7.3.

Example 7.3

In figure 7.11 below, $\Delta ABE \sim \Delta CDE$.

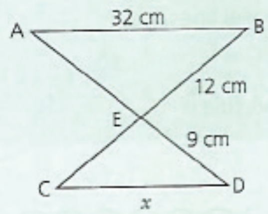


Fig 7.11

Calculate the length of CD.

Solution

Since $\Delta ABE \sim \Delta CDE$, the ratios of the corresponding sides are equal.

That is, $\frac{CD}{AB} = \frac{DE}{BE} = \frac{CE}{AE}$

$$\therefore \underbrace{\frac{x}{32} = \frac{9}{12}} = \frac{CE}{AE}$$

$$x = \frac{9}{12} \times 32 = 24 \text{ cm}$$

$\therefore$ CD = 24 cm.

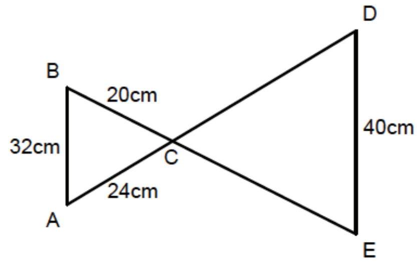
In certain cases, it is possible to show that given triangles are similar even when the values of the corresponding sides and angles are not indicated.

The information provided in this example implies that $\overline{BE}$ and $\overline{DE}$ are corresponding sides. They are not. This error is the main source of the misconception that may be carried forward by a learner reading this example (textbook) as their only source of knowledge on this topic. The author went on and calculated the length and gave the wrong answer. It is possible that the error originated from naming similar triangles: $\Delta ABE \sim \Delta CDE$. When naming similar triangles we must consider congruent angels and corresponding sides. In this case, assuming $\overline{AB}$ and $\overline{CD}$ are parallel:

$E\hat{A}B = E\hat{D}C$ (Alternating interior angles)
$A\hat{B}E = D\hat{C}E$ (Alternating interior angles)
$A\hat{E}B = D\hat{E}C$ (Opposite angles or third angle of a triangle)
Therefore the correct naming of these similar triangles is $\Delta EAB \sim \Delta EDC$ or $\Delta ABE \sim \Delta DCE$ or $\Delta AEB \sim \Delta DEC$. In this way corresponding sides are not missed. For example, $\overline{BE}$ and $\overline{CE}$ are corresponding sides.
The misconception picked by students was noticed when students were given the following question in a test:

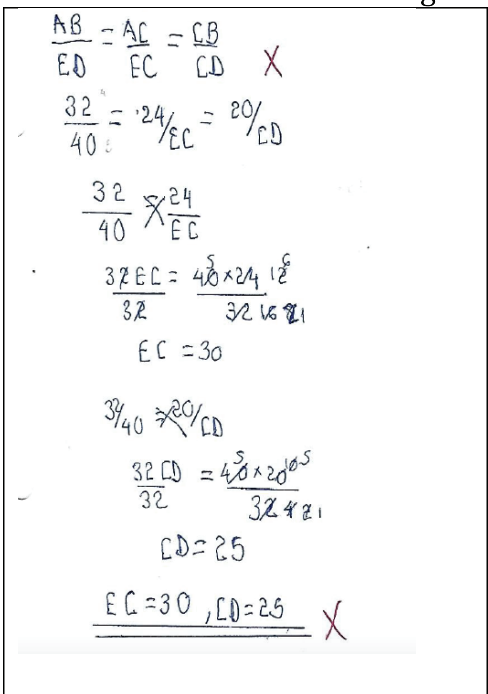
Use the figure below to:
(a) Prove that ΔABC is similar to ΔEDC

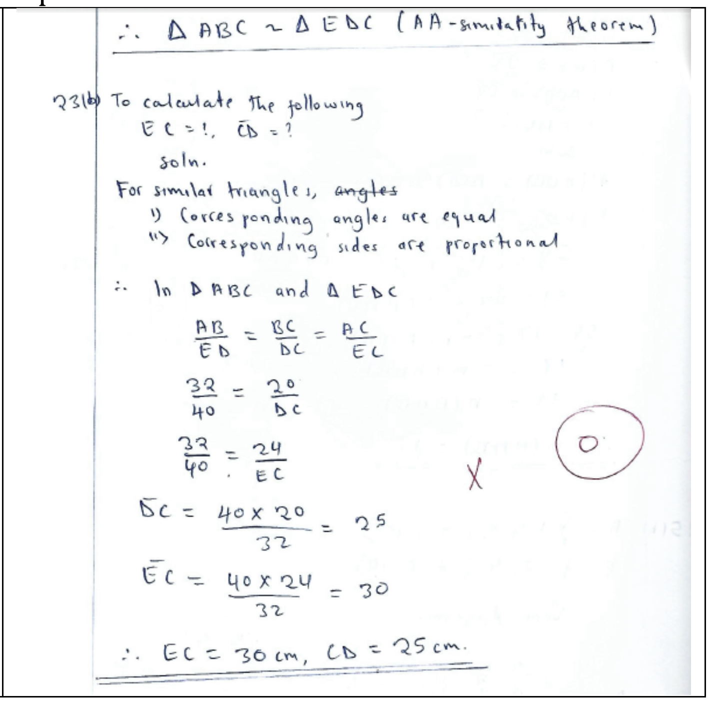
(b) Calculate the length of EC and CD.



Source: Form 2 National Examinations 2004, Q 24.

Two of the fifteen wrong attempts are:





The mistakes made by these students are similar, and consistent with the errors made in the example given in the textbook.

Case 3: Form 3 textbook, page 9, Example 1.13.

Example 1.13 If R = { (x, y): $y = \sqrt{2x-1}$ } where x and y are real numbers, find the domain and range of R. ***Solution*** Domain = All elements of the first set (x) which make the relation possible for real numbers. For any real number the square root of negative numbers do not exist, therefore: $2x - 1 \geq 0$ $2x \geq 1$ $x \geq \frac{1}{2}$ $\therefore$ domain = { $x : x \geq \frac{1}{2}$ } To gate the range; make x the subject of the equation $y = \sqrt{2x-1}$. $y = \sqrt{2x-1}$ means that $y \geq 0$. $y^2 = 2x - 1$ (square both sides) $2x - 1 = y^2$ $2x = y^2 + 1$ $x = \frac{y^2+1}{2}$ For all real numbers y, x is defined but it is restricted to $y \geq 0$. $\therefore$ range = { $y : y \geq 0$ }	Although both domain and range are correctly given, the method used to find the range (making the independent variable the subject), and the ambiguity in the conclusion "…. x is defined but it is restricted to $y \geq 0$" may have contributed to students' misconceptions. The comment "$y = \sqrt{2x-1}$ means $y \geq 0$" is enough to determine the range of the given function.

Although this example was later discussed in class and the confusion clarified, students went on to make the same mistake in the test.
The problem in the test was to find the domain and range of $y = \sqrt{x+4}$. Seven out of twenty students gave solutions that are equivalent to the two solutions below.

Range = ? $y = \sqrt{x+4}$ $x + 4 = y^2$ $x = y^2 - 4$ $\therefore$ Range = { $y : y \in \mathbb{R}$ }	Range. Values of y that give x. $y = \sqrt{x+4}$ $x = y^2 - 4$ $\therefore y = \in \mathbb{R}$ $\therefore$ Range = { $y : y \in \mathbb{R}$ }

This problem is very similar to the example solved in the textbook. Both students used the method in the textbook, making x the subject. In fact this is the way their teacher presented the concept the first time in class. In the above problem, the method led them to the wrong conclusion. As seen in the equation formed, x is defined for all real numbers y, but this is not the range of the given function. The fact that seven students went on to make similar mistakes after the misconception were cleared implies that the confusion was greater.

Concept/topic	Number of students who made mistakes (in the test or homework) before the error was corrected	Number of students who made mistakes (in the test) after the error was corrected
Solving inequalities (case 1)	10 (out of 15)	3 (out of 15)
Similar triangles (case 2)	15 (out of 15)	0 (out of 15)
Range of a function (case 3)	20 (out of 20)	7 (out of 20)

Table 1: Frequencies in the study

Conclusions and Recommendations

In a country where schools are too full like Tanzania, many students rely on textbooks for learning. Errors, contradictions, ambiguities or confusion in textbooks contribute heavily to the continuous poor performance in examinations. The books have been in circulation since 2009, implying that some current mathematics teachers used the same textbooks to learn and now they using them to teach. The errors are alarming, they cannot be ignored. Tanzania Institute of Education (TIE) and authors of textbooks should consult subject experts to check textbooks before authorization and distribution.

Teachers are advised to prepare well before going to teach, use multiple ways of representing solutions and give students opportunities to comprehend what they have learnt. Doing this helps in spoting mistakes in the textbooks. Also, teach in a way that allows students to think, because if they do, they may help notice the errors in their textbooks. For example, solving inequalities involving absolute values may be introduced by helping students to understand that:

$|z| < b, b > 0$ is equivalent to $-b < z < b$, and

$|w| > d, d > 0$ is equivalent to $-w < -d$ **or** $w > d$.

In this way, (and not simply $\pm z < b$ or $\pm w > d$ as seen above and in their exercise books) students then choose how to present their solutions and hence enhance their thinking and understanding.

In addition, it is a good idea to provide answers to exercises in textbooks. Checking their answers, students may notice when errors are made. Also, answers to exercises will help students to confirm their solutions and hence develop confidence in what they have learned or what they know rather than waiting for their teacher(s) to mark their work.

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The role of written feedback in enhancing students' mathematics learning in Tanzanian lower secondary schools: an intervention study at a lower secondary school in Bukoba municipality, Tanzania.

Evodius Jackson

Almuntazir schools

This study examines the roles of written feedback in enhancing students' mathematics learning in Tanzanian lower Secondary Schools. An intervention study was conducted in form one and two classes using sequential mixed methods. Two mathematics teachers, one from each class participated in the study. The study was guided by three research questions and data were gathered in three phases, before, during and after the intervention. The data sources were document analysis, questionnaires, observation and interviews. The findings before the intervention indicated that written feedback provided did not promote much of students' learning. Reasons identified for not giving good feedback include; inadequate teachers' skills, large number of students and heavy teaching load. Findings indicated that during the intervention teachers were equipped with skills and effective written feedbacks were developed. Data revealed that feedbacks were helpful to students' mathematics learning. The findings of this study suggest that teachers need to undergo regular in-service training on how to conduct formative assessment, including provision of effective written feedbacks. The key further research is recommended on how students can be effectively trained to conduct peer assessment, including giving feedback.

BACKGROUND OF THE STUDY

One of the very important stages in teaching and learning is assessment. It is through this
Stage both teacher and student's progress is determined. Here, the assessment helps to elicit the evidences of learning by determining how much skills and competences have been developed. How students are assessed and how the evidences gathered are communicated to them, matters a lot. This matters in the sense that, the information communicated through feedback conveys the evidences of the student‟s performance, so if properly presented (to students) can promote student‟ learning.

One of the ways in which teachers present the evidences of students‟ learning progress is through providing them with written feedback. Compare to other forms of feedback, through written feedback teachers spend a considerable amount of time to pass though students‟ works and, presumably, ways to improve students‟ learning are developed. Additionally, the written feedback is documented so students can use them to re-learn.

The literature and my own experience as the student and a teacher support that, a well constructed written feedback stimulates and encourages students‟ mathematics learning. As I was a student, when a mathematics teacher gave back the marked examination paper, I was very anxious to look at the grade and comments a teacher had made. I was highly motivated when the teacher appreciated my efforts and identified weakness points and suggested the ways which I could follow to correct errors. On the other hand I was disappointed when I received a paper with crosses without identifying errors and proposed ways correct them. Mikre (2010) argues that, teachers are responsible for providing detailed descriptive feedback which will guide students to re-learn and meet learning objectives.

The study intended to close the gap reported by Lee & Schaller (2008) that most of teachers assume that ticks and cross are the meaningful feedback and that they assess students for the purpose of ranking them and not to monitor their learning progresses. The other gap which this study intended to close is what is argued by Black & Wiliam (2004) that students ignore comments and focus on the grades.

The findings from Kyaruzi (2011) revealed that teachers (in Tanzania) lack enough skills for assessing and provide students with effective feedback, similarly Ndalichako (n.d.) revealed that some teachers in Tanzanian secondary schools use assessment to punish students.

STUDY CONTEXT

An intervention study was carried out in one public secondary school in Bukoba Municipality. Kaloli secondary school is situated at the center of Bukoba town. It is a mixed school with lower and upper secondary classes. The school has both day and boarding scholars. The school became the researcher's choice since it had at least one mathematics teacher in each class and therefore this provided an opportunity for the researcher to work with teachers within their normal classrooms without interfering other classes' time table. The school has 6 mathematics teachers; the total number of teachers in this school is 40. Each class (form one to form four) is divided in to four streams and two stream in each form five and six classes, each stream has at least 60 students.

RESEARCH PARTICIPANTS

The study was conducted in a form one and two classes, the classes had 60 and 50 students respectively. During the intervention all students in both classes were provided with written feedback and then a total of 14 students, 7 from each class were selected as a sample for interview. The study also enrolled two mathematics teachers, one from each class.

SAMPLE AND SAMPLING PROCESS

14 students were selected, 7 females and 7 males. These students were stratified in terms of their mathematics performance. The performance records were gathered (during document analysis) from the records of previous assessment which included the monthly tests and terminal exams. The high and low achievers were those whose scores ranged from 80% to 95% and 10% to 40% respectively. This based on the general performance which was observed ranging from 10% as the poorest mark of students and 95% as the highest mark of students.

RESEARCH DESIGN AND METHODOLOGY

The research questions being investigated are:

1. What written feedback do students receive?
2. What factors determine the quantity and quality of teachers‟ written feedback?
3. What can the influence of mathematics written feedback be to students?

The intervention research formulated the framework of this study. The approach was appropriate to this study since it aimed at working with mathematics teachers in planning and providing written feedback which could enhance students' mathematics learning.

RESEARCH METHODS

The study used sequential mixed design where by both quantitative and qualitative and variety of tools such as document analysis, questionnaires and interviews were used.

DATA COLLECTION METHODS

The data was gathered in three phases which are; before, during and after the intervention.

Document analysis: The documents checked include students' mathematics exercise books and marked examination papers which had been returned to students. Photographs of the written feedback were taken for further examination and analysis. The information gathered from these documents pave a way for the researcher and mathematics teachers to plan for the modified feedback practice which would enhance students‟ learning.

Questionnaire: The closed questionnaires (written in Kiswahili) were provided to 14 students who filled and returned them before the intervention. The reason for students to return the filled questionnaires before the intervention was to gather the students‟ perceptions and experience with the existing written feedback in mathematics so that the researcher and teachers would plan and give good written feedback.

Observation: A total of 8 lessons were conducted, 4 lessons for each class. The duration for a single lesson was 80 minutes, 40 minutes used for presenting and other 40 minutes for giving written feedback. Students were observed on how they would react to written feedback.
Face to face Interviews: Interviews for the two teachers were conducted before and after the intervention while students' interviews were conducted after the intervention, during the interview students were asked to bring their exercise books.

DATA ANALYSIS

The study was an iterative one, the findings in one iteration led to the plan for the stage that followed, so the process of data analysis began as the data collection started. For example the data collected from document analysis and questionnaires were examined and analyzed, the results were used by the researcher and teachers to set action plan for giving written feedback provided during the intervention.
The quantitative data (questionnaires) were analyzed using the frequency distribution (histograms in excel).
The audio data were transcribed and then coded, and errors were checked, finally data were combined with other data gathered via other methods. Furthermore, the data were transformed in to categorized themes.

FINDINGS

Before the intervention:

Both students and teachers reported that they know the importance of feedback in learning.
Availability of written feedback

- The most given written feedback were ticks and crosses, with no guiding information.
- Availability of warning comments such as "*be careful*" and "*don't cheat*". Motivation comments (e.g. "*very good" and* "Excellent" given only to students who could answer all question provided correctly.
- Many questions (e.g. 15 questions) provided though some not marked.

Reading and understanding of written feedback

Students reported that they are motivated by positive feedback; however teacher revealed that most of the students prefer grades than written comments.
Challenges in providing effective written feedback
Teachers reported about the challenges in giving good written feedback, these include; large number of students, heavy teaching load and time allocated.

DURING THE INTERVENTION

A feedback plan was developed to guide provision of written feedback, the following are the characteristics of the plan:

- use of terminologies and handwriting which students could read and understand.

- focus on the intended learning area.
- detailed information which specifies where an error has occurred and suggests what has to be done.
- give feedback on time to allow immediate learning.
- encourage student"s self-regulation.
- encourage student"s thinking, e.g. ask more questions etc.
- use of more specific positive/motivation/praise written comments.

Descriptive written feedback was given but with no numerical grades, then the marked papers were returned to respective students and asked to grade themselves in %.The papers were re-submitted and then the students' proposed grades were matched with the teacher's grades. Each session (lesson) was analyzed by looking at the written feedback provided and their impact on students' leaning. The analysis of each lesson could help to adjust and make improvement in the lessons that followed. According to teachers, as the process took place there was an increased participation of students as more exercise books were collected for marking than before, as they said - not all students could submit their exercise books before.
14 students asked to grade their works, basing on the written feedback provided - 5 students graded themselves exactly as the teacher did, while the grades of 4 and 5 students were less and more than the teacher's grades respectively.AFTER THE INTERVENTION

The intervention was conducted for four weeks, then after 14 students and two teachers were interviewed so as to collect their views about the written feedback provided.

The data collected from the interview show that teachers gained knowledge of giving effective written feedback to students as the result there was an increased participation of students in responding to feedback plus the rate of collecting their exercise books.

The data reveals that students were able to read the comments/feedback provided; they were able to share the feedback to friends and parents. Generally, students reported that the written feedback encouraged them to think and learn more, furthermore, this created friendly learning environment with their teachers.

IIMPLICATION OF FINDINGS

The data collected before the intervention reveals that teachers provided written feedback, the most used feedback were ticks and crosses; much of the feedback could not enable students to identify areas of weakness and proposed ways to improve.

The data shows that teachers are aware that feedback are important information which would motivate students' learning, however their understanding does not match with the kind of feedback they provided, for instance, some of the children could not collect their exercise books and scared to share the written comments to friends and parents. The feedback plan developed and increased the ability of teachers to give written feedback.

The increase number of students to do classroom and home tasks/questions and an increased number of exercise books collected can be interpreted that the written feedback given during the study encouraged more students to participate in learning. Furthermore, this enabled them to share their learning experience to parents, and as stakeholders, parents would be motivated to support children's mathematics learning.

The study reveals the need for teachers to undergo regular in-service training on how to conduct effective formative assessment, including giving effective written feedback, this would be reached through the support from the school and other stakeholders.

REVIEWED FINDINGS

Basing on the teaching and learning practices conducted by the researcher at current working station, new recommendations are placed.

Parents' perceptions about written feedback

At his working station the writer works with fellow teachers to encourage assessment for learning, one of the areas emphasized is the use of feedback to improve students' mathematics learning where by written feedback given promotes thinking and encourage students to identify their errors and correct them. From here, the complains raised from parents who claimed that the teachers should be responsible to correct errors and not to let children correct the errors by themselves.

This led to the call for parents and teachers' meeting, from the meeting the parents were informed about the importance of letting students realize their mistakes and then make them take control of their learning by correcting their errors. It was put clear that through these comments students would think and learn more.

There are different ways in which parents interpret the role of the teacher and students in teaching and learning. It was realized that most of parents think that teachers must do everything for children to perform well, however, after the clarification from teachers, parents supported the approach.

Number of tasks/questions given to students

One of the challenges mentioned by mathematics teachers at Kaloli secondary is large number of students in the classrooms, this stopped them from giving constructive written feedback to all students, however, it was observed that teachers provided unreasonable (too many)number of questions which they were not able to assess.

Currently, through various professional development program, a plan has been set that reasonable number of questions should be given to students, this number allows teacher to assess learning at each individual student. Through this approach, more improved assessment of students' learning has been observed and parents have appreciated.

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What are the relevant assessment techniques in mathematics in the context of competency-based curriculum?

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Abstract

Between 2004 and 2008, Tanzania reviewed school curricula at all levels of education. The review was aimed at shifting the teaching and learning paradigm from content-based to competency-based. There were a number of factors that necessitated this shift. One of the major factors was that graduates from what was considered as content-based curriculum could not demonstrate the competencies that could help them cope with local, national and global market demands. Thus the reviewed curriculum was aimed at enabling graduates acquire competencies in addressing current and future national goals, global demands and challenges in ever changing human needs. Besides, the revised curriculum emphasised teaching effectiveness in the use of interactive, participatory teaching and learning approaches and child friendly environment. In the curricula, these six areas of competency were emphasised, that is, communication; numeracy; creativity and critical thinking; technology, interpersonal relationships; and independent learning. After reviewing the curriculum, efforts were made by the Government, specifically the Ministry responsible for education to orient teachers and other education officials on how to implementation the new curriculum. Major emphasis was put on teaching methodological skills and challenging or rather difficult topics in different subjects, including mathematics. Assessment as one of the important processes in teaching and learning was not given due weight. Paper-and-pencil tests/exams continued to dominate assessment procedures. This paper will bring to light what are perceived to be relevant assessment techniques in mathematics in the context of competency-based curriculum.

Assessing teaching and learning of mathematics in a grade four multilingual classroom, in Zambia

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A major social characteristic of Zambia is that it is multilingual, multiethnic and multicultural.

There are reportedly 73 languages in Zambia with seven major languages. The diversity of ethnicgroups with their related languages has led to the existence of several traditions and cultural practices which have implications on the education of children (Tambulukani, 2015). *The Ministry of Education (2013)* introduced the *use of familiar language as a medium of instruction in primary schools* from grade one to four. *The current study assessed the teaching and learning of mathematics where a local language was in use in a multilingual class in selected primary schools in Lusaka district, Zambia. The study focused on teachers' practices as well as learners' as they engaged on mathematical activities in a multilingual classroom. What happens in class to account for the development of mathematical and pedagogical practices in multilingual contexts? We thus sought to establish how learners coped or chose or made meaning as they endeavoured to participate in the mathematical activities and what strategies teachers used to ensure mathematical knowledge acquisition. The research was carried out in two primary schools. The target population was teachers and pupils in the 4th grade whose mother- tongue was not the language of instruction. The research was qualitative in nature and it took the form of grounded interpretative classroom research. The lesson observations and recordings served as empirical basis of the results. The preliminary results show that teachers tended to use familiar 'language' among the learners – a language that was a mixture of English, the dominant language of the area and other common terms of languages in the country. The participation –active or of hesitation- among learners appeared to depend on the prevalence of the terms in the discourse. The linguistic shaping of the instruction by teachers was supplemented by use of visual displays,*

gestures, body language etc which appeared to form common ground for learners for establishing mathematical concepts. There were opportunities and challenges but what appeared to come through was the delay in establishing 'academic mathematics terms' or for the majority of the learners to 'speak mathematically'– Pimm (1987), that is to become proficient in a mathematics register and in this way to be able to act verbally like native speakers of mathematics.

Assessment of grade 1 learners' pre-school knowledge in Lesotho

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This paper aims at addressing two questions: How can we assess what Basotho Grade 1 learners know and can do at the start of school? What are the complexities of adopting assessment tools developed in a different context to that of Lesotho? Assessment of pre-school knowledge had not been part of the educational agenda in Lesotho for decades. Hence the purpose of the study reported in this paper was to assess pre-school knowledge that Grade 1 learners bring to Lesotho primary schools. The first year of school is an important phase of a learner's education and effective educational provision during Grade 1 is associated with later outcomes right up to the end of secondary school (Tymms, Merrell & Bailey, 2017). Providing Grade 1 teachers with information about the abilities of learners can help them to target their teaching at an appropriate level.

In order for us to assess learners' knowledge when they begin school we are using the International Performance Indicators in Primary Schools (iPIPS; www.ipis.org) framework (Tymms et al 2017). iPIPS is a unique international monitoring system for children starting school. It is a model developed at Durham University, UK. Given that iPIPS is developed in a different culture and language to that of Basotho, we had to adapt them to suit the local context. This process involved a cultural review and a translation from English to Sesotho. There are complexities associated with translating technical tools from English to Sesotho. For example, rhyming words in English could not find equivalent ones in Sesotho because rhyming does not exist in the local language. As a result, innovative solutions had to be found. In the numeracy section, phrases like "what is twice three doubled?" was very hard and it was difficult to translate. Some words were unfamiliar such as violin, yacht and saxophone to a Mosotho child. It is therefore recommended that during translation, technical words be left in English. At the policy level, it is hoped that knowing what children bring to school might shape teaching and learning in Lesotho primary schools.

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Diagnostic assessment in mathematics: the case of an entry-level cohort in an engineering department

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This study presents a motivation for, and an analysis of the data arising from, the diagnostic assessment of a cohort of engineering students at a South African university of technology. This study reviews the mathematical underpreparedness or otherwise of many first time entering university students in Engineering. Misalignment between their school mathematics and what is required in a first year university mathematics course is the core problem which is the subject of this study. The diagnostic test used in the study is the standard national test for post school leavers called the National Benchmark Test. Generalised mathematical competencies, that is algebraic processes, functions, concepts in trigonometry, transformations and spatial perceptions are the areas targeted for assessment. Findings arising from an analysis of the data indicate serious misalignment of competencies as well general strengths and weaknesses of both whole groups and individual candidates. The implications of the findings are of value both at school level and in first year courses. Implications for the university programme point towards curriculum intervention, including adaptations to pacing, streaming and teaching and assessment approaches. Targeted technological intervention is also recommended.

Sub-theme 7

The role of contextually relevant research in quality mathematics education

Introducing the strangers: mathematics teacher leaders and their roles in the professional learning context in Tanzania

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This paper explores the roles that mathematics teacher leaders (MTLs) assume in their work of leading the professional learning of mathematics teachers in rural and remote communities in Tanzania. Shaped by notions of symbolic interactionism, this multi-site case study drew eight participants who, for the first time in their professional lives, served as leaders of the professional learning of mathematics teachers in their school district. Research data were gathered through in-depth interviews and vignettes constructed by the leaders. The roles, greeter, introducer, walker, distributor and provoker, motivator, and facilitator were carefully uncovered from within- and cross-case analyses.

INTRODUCTION

Our study aimed at developing a deeper understanding of the roles that MTLs assume while leading the professional learning of mathematics teachers. The MTLs in this study are primary school mathematics teachers who facilitated the professional learning of other mathematics teachers in rural and remote communities in Tanzania over three years. The notion of a primary school teacher serving as a leader of mathematics teacher learning is innovative within Tanzania. In 2015, Hardman et. al. found that was a traditional view of teacher learning in Tanzania. Teachers were to assume a passive role in the process where they were seen as recipients of knowledge rather than active constructors of knowledge. The leaders of professional learning were often 'outsiders' to the primary school classroom; that is they were representatives of government agencies or NGOs.

METHODOLOGY & THEORETICAL FRAMING

This qualitative multi-site case study (Stake, 2006; Lauckner et al., 2012) is framed around notions of symbolic interactionism (Blumer, 1969) that suggests that human beings develop meanings out of their interaction with the situation or event through interpretative processes and through social interactions.

Eight MTLs (three females; five males) working in different rural primary schools and in different districts participated in this study and were each considered a case in the research design. Rural schools are those schools located in areas that are characterized by "poor infrastructure and poor social services" (United Republic of Tanzania, 2010, p. 17).

Each MTL held a Grade A teaching certificate and were a part of the Capacity Development for Mathematics Teachers in Rural and Remote Communities in

Tanzania 5 year project, funded by Global Affairs Canada (GAC) (Simmt et al., 2011). The primary school mathematics teachers were identified by their districts as master teachers and were subsequently identified by their peers in the short course to serve as leaders of professional learning. The short courses, held over 4 years, were designed to develop participants' professional knowledge and skills around mathematical concepts, inclusive pedagogy, and models of professional learning. Three of the MTLs had less then 10 years teaching experience. Five of the MTLs had 20 or more years of teaching experience.

Data for the study were gathered through: in-depth, one hour, interviews and vignettes (Lieberman, 1987; Lieberman & Friendrich, 2010) that were constructed by each of the participants. We used what Miles and Huberman (1994) called, "concurrent flows of activity: data reduction [and] data display" (p. 10) for within- and cross-case analysis.

RESULTS AND DISCUSSION

Six roles were assumed by MTLs whilst leading professional learning of mathematics teachers. We share illustrative examples of each role in this section.

Greeter

All of the MTLs indicated engaging in greeting mathematics teachers. The greetings were the first thing they should do before initiating the learning process. One leader, described:

> Yes, so, we normally use the *Hisabati ni Maisha* saying to greet the teachers. And it has been a greeting that I have decided to adopt in my mathematics classes in my school. Okay, so when I just step in, I just say *hisabatiiiiii!* (mathematics) and the teachers will respond … *ni maishaaaaaa!* (is life) I can even repeat it twice as much as I think I need to do so. But … it's not like our schools where students must stand up to reply to the greeting, and teachers normally remain seated.

The phrase, *Hisabati ni Maisha* (Mathematics is Life; Mathematics is Living), was used in the short courses and was a slogan for the GAC project. The decision of using the slogan was inspired by the leader's intention of making their learning sessions to sound mathematical from the onset, considering that the phrase starts with mathematics. The MTLs observed the phrase to take mathematics teachers back to what they are expected to do as they work to enrich their professional knowledge and skills. The use of a greeting, as a cultural dimension, and can help teacher leaders connect with the teachers as "part of the context and the setting in which ... [teacher professional learning] operates" (Hord & Sommers, 2008, p. 49).

Introducer

The participating MTLs reported the importance of introducing themselves and their teacher colleagues. For them, the introductions are meant to enable members of the groups to become aware of each other before engaging in a collaborative learning practice. As one leader expressed, the introductions were "something that we must do when teachers convene for their [professional] learning." He added further that introductions of teachers are indispensable because of experiencing attendance of new teachers who join the sessions for the first time. Given such a situation, "teachers needed to understand their colleagues for them to feel comfortable in their new environment." It was also important for the leader to introduce themselves, as it was important for mathematics teachers to realize that their leaders were primary school teachers who knew what it was like to teach mathematics in rural and remote primary schools.

Distributor and Provoker

To initiate the process of teacher professional learning, the teacher leaders reported embarking on distributing learning tasks. One leader, documented in her vignette that she "provided the tasks to teachers for them [teachers] to start doing in pairs first, then in small groups, thereafter to be discussed in the large group." Her intent with such a style of distributing tasks was to ensure that every mathematics teacher work on the tasks, so they become actively engaged in their learning right from the beginning of the session. The MTLs also used the tasks to provoke teachers to share their understanding of the concepts they were learning in their sessions, that is, to provoke the teachers to communicate their thoughts related to the concepts. This provocation was another way for the leaders to invite the teachers to take an active role. The MTLs' use of the tasks resonate with Murray and Zoul's (2015) views that teacher leaders should use learning tasks to find "where each of our teachers is in their own learning journey and where they think they need to go next in order to grow and improve" (p. 11).

Walker

The MTLs reported walking around to meet and talk with mathematics teachers about what they are experiencing in their learning. The decision, as most of them said, was motivated by the need to realize opportunities for helping teachers to focus on improving their professional knowledge. One leader reported that engaging in gathering information about the learning of the teachers during the sessions by comparing "what is happening" in every group. He decided to pursue such a direction in the interest of identifying groups that were well-positioned to support learning in other groups, which were experiencing some difficulty. For another leader, the movement was important to "listen to the teachers in relation to the challenges that they face" as learners. The commitment shown by the MTLs reflects the suggestion made by Hord and Sommers (2008) about what a teacher leader can do to continuously monitor the professional learning of teachers. The teacher learning scholars note that "there will be bumps, dips, and detours on the road … [so] someone must monitor the pulse of individuals … to help ease them over the rough spots" (p. 114).

Motivator

Most of the MTLs described verbal motivation and body language to encourage their colleagues to engage in learning and to navigate their own professional learning journey. One leader found motivating mathematics teachers a vital role especially because his colleagues were in the midst of a new professional learning experience. With such a consciousness, he comfortably pursued the role of heartening his colleagues to keep working on advancing their professional knowledge and skills. As he described, his commitment was shaped by his desire of not seeing mathematics teachers lose their industriousness during their learning nor did he intend to leave them exhausted to learn. He considered himself responsible for making sure that “the teachers are motivated to learn [amidst] tensions and challenges that might discourage them to learn.”

Facilitator

Nearly all the MTLs reported feeling responsible for creating conditions for productive discussions among mathematics teachers during sessions. It became clear to us that the MTLs decided to assume such a role because they wanted to make sure that every mathematics teacher was engaged in the professional learning by sharing ideas and experiences regarding the concept under discussion. As one leader explained in an interview:

> Yes! What I learned during our preparatory program [short course] is to make sure that every teacher is engaged in the learning process. So, it was important for me to make sure that everyone realizes a chance to participate in learning. The idea was to make sure that everyone there is having an opportunity to be heard if she or he wants to do so.

Louis et. al. (2017) observed that teacher leaders are expected to take a leading role in creating supportive systems and conditions for teachers to actively further their professional knowledge.

CONCLUSION

Our study has described leadership roles that a mathematics teacher leader assumed when leading the professional learning of mathematics teachers in rural communities in Tanzania. We conclude that a description of the MTLs’ roles during the professional learning of mathematics teachers is an opportunity to rethink teacher professional learning practices and how it can be situated in the hearts, hands, minds, and bodies of teachers themselves.

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An exploratory study of teachers' experiences of professional development (pd) course in South Africa

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The focus of this paper is the Straussian grounded theory analysis of pre-teaching interviews of two participants in a PD course. We report on the teachers' initial thoughts about the benefits of the Transition Maths1 course, in particular the influence of the Mathematic Teaching Framework (MTF), a professional development resource offered in the course, in their practice. Initial results suggest that teachers' motivation to learn and change influenced their take-up of PD.

INTRODUCTION

The introduction of Curriculum and Assessment Policy Statements (CAPS) and its attendant documents in 2012 in South Africa introduced a new set of demands for mathematics educators. The accompanying annual teaching plan (ATP) and more recently scripted lesson plans suggest that teachers don't have the expertise to design their own learning programs and must therefore adhere to what obtains in these prescribed documents and other resources supplied by the national of provincial Department (Ramatlapana and Makonye, 2012). However, these resources are not without problems, and thus ultimately depend on how teachers are able to use them where appropriate, or adapt. This is particularly the case in contexts where poor learner performance persists, and has been linked to the quality of mathematics teaching, as is the case in South Africa. Consequently, continuous in-service professional development of teachers is a productive enterprise to enhance teachers' mathematics knowledge for teaching (e.g. Hoover, Mosvold, Ball & Lai, 2016). Against this setting, the Wits Maths Connect Secondary (WMCS) project and the Transition Mathematics 1 (TM1) course in it developed a teaching framework to guide their work with teachers on their mathematics teaching. Mathematics Teaching Framework (MTF), is an adaptation of an analytic framework - Mathematics Discourse in Instruction (MDI) - developed for research in the wider project. The MDI Framework was developed to describe and interpret differences in mathematics teaching (Adler and Ronda, 2015), and its elements selected so that the mathematics made available to learn in a lesson was in the foreground. The Maths Teaching Framework (MTF) as an adaptation of the MDI is a planning and reflection tool for teaching offered to teachers in the PD, through which they can deliberately plan and reflect on their lessons, and particularly on the mathematics they would like their learners to come to know and be able to do. In the broader study, the emerging stories and practices of four purposefully selected teachers, all of whom have participated in the course, will be use to explore the influence of the framework in their practice and thus theorise their take-up. The different interacting elements of the MDI framework for teaching, will structure the analysis of their lessons the main focus of which is on what mathematics teachers made available to learners in their teaching in the broader study. These are complemented by pre- and post-observation in depth semi-structured interviews that invite teachers to describe their experiences of teaching, and thus tell stories about their participation in the TM1 course. The pre-interviews of two teachers are the focus of this paper.

Theoretical Grounding

Neo-Vygotskian activity theory is used to ground the broader qualitative study. Activity theory is a particularly useful lens as it provides the means of understanding and analyzing take up of PD and affords the means to find patterns and construct interpretations across interactions, particularly in relation to the MTF as a boundary object. However, despite its strength in describing and presenting phenomena with its built-in shared language, we are mindful of imposing the theory on the data rather and thus preventing the data, and so the teachers' stories, to inform the theory. This is particularly important as teachers move from the course into their classroom contexts. For this reason, we have adopted Straussian grounded theory to ensure that the data rather than an existing theory structure the analyses and the subsequent interpretations and explanations. In this vein the analyses of the pre and post interview will be strictly through Strauss and Corbin (1998) grounded theory; that is open coding followed by axial coding.

The MDI framework informs teaching of both mathematics and mathematics teaching in the Transition Maths 1 course. Based broadly in socio-cultural theory, MDI is not at odds with the framing of my study with neo-Vygostkian cultural historical activity theory (CHAT). As a research tool, the MDI framework functions as a tool to study the nature and quality of mathematics made available in episodes of teaching across a lesson. It will be used to analyse lessons in the wider study.

Participants, data, and analysis

The broader study will explore teachers stories of their experiences using a grounded analysis of semi-structured interviews with teachers, following their participation in the PD and prior to the observation of their teaching, and then again after the observation. We will use the classroom data and pre and post teaching interviews as a means of triangulating teachers' stories of take-up of PD. In this paper, the focus of the analysis will be the initial pre-teaching interviews of two out of the four cases in the broader study. One of the teachers, male, has a Bachelors in Chemical Engineering and the other, female, has Bachelors in Mathematics. Both teachers have completed Honors in Secondary Mathematics Education in order to launch their teaching careers. Both have been teaching for less than 5 years and were interested in enhancing their mathematics knowledge for teaching. The similarities in their personal and academic profiles motivated their inclusion in this paper. The pre-teaching interviews consisted of two parts. The first part, which is the focus of this paper, is designed to elicit from teachers renewed insights they might have developed on teaching and mathematics through participation in the course. The second part, broadly structured around the components of the MTF, is designed to elicit from teachers the new practices they have established around the four components and the challenges involved. **S**elected responses highlighting the coding of selected responses of the first half of the pre-teaching interviews are featured in this section. The coding process proceeded through open coding, exemplified in the table**s** 1, 2 and 3. Keywords from the transcripts of teachers were used to label and identify the essence of what the two cases were saying. For example when asked about what they considered to be the main take away from the course (see table 1). Both cases approached it in a similar manner. They foregrounded what their struggles were before enrolling into the course. For Thandi, she was discouraged by the amount work she had to do, a problem that was further compounded by negative mindset of learners. Attending the course gave her a sense of hope and provided guidance as to what she could to improve the situation. For Rolinga, his concerns were on not identifying what was expected of him as a teacher. The application of policy was a challenge prior to enrollment in the course. However, the course provided him the means to understand what is expected of teachers in relation to policy, its application and connection to classroom practices. The axial coding, the second phase of the coding process, involved putting the data that has been fragmented in the opening coding process back together into a coherent story of their experiences at PD using six broad categories identified by Strauss and Corbin (1998). Thandi's and Rolinga's responses were classified as context, central phenomenon, causal conditions, intervening conditions, actions/strategies, and outcomes. In Table 1 below, the intervening conditions for both cases are opportunity to enroll in the Transition Maths 1 course. The context for Thandi was the challenge of managing the tasks of teaching and learner characteristics. In Rolinga's case, it was lacking the means to align policy expectations of teachers with his classroom practice. Thandi cited developing a sense

Raw Data	Opening Coding
Thandi: I started teaching last year; I was very **discouraged** when I came into the field…There was **a lot that needed to be done**… The kids come with **negative mindsets**… a lot of foundation which they did not receive. So it was very overwhelming… But then when I started attending the course, it gave me a **sense of hope**. Because I started to see things differently. I started to analyse what it is I could be doing to improve ...	Discouraged Negative mindset of learners Sense of hope
Rolinga: The key things that really came into focus are the **issues of curriculum** …actually mathematics it was never really a challenge for me, but … in terms of the ATP,… we just apply the document more than anything else… the policy is there but the **application of the policy that was the challenge**. So the course actually helped us to say **what is expected of us** in terms of when we go to class… Now I have the **mechanism**…I have the skills on how **to deliver … what is in the curriculum…**	Issues of the curriculum Applying policy a challenge mechanism to deliver the curriculum

of hope, while Rolinga identified understanding policy and what it means for teachers in terms of classroom practice as the outcome for

participating in the course. Illustrations of the remaining categories will be provided as

more excerpts are added in the discussions.

In response to questions regarding how their participation in the course has changed the way they think about maths and teaching mathematics (see tables 2 and 3 respectively), the two cases reported changes relating to mathematics knowledge for teaching and its bearing on core practices teaching; that is explanation, exemplification and facilitating learner participation respectively – all key elements of the MTF framework. Thandi's response on how she thinks about mathematics revealed a very personal journey in the sense that participation in the course forced her to rethink her mathematics competence and focused from thereon, on deepening her subject matter knowledge.

Raw Data	Opening Coding
Thandi: For me **I always thought I knew mathematics**, you know, coming from a BSc. So but then, the first two courses especially the first two lessons, they made me think differently because I could not figure out some of the questions. …for example the use of Geogebra, Especially with Trig transformation. I saw a lot of challenges with that. So the use of technology helped me to **understand the maths content**.	Thinking differently about her content knowledge Use of technology to understand Trig
Rolinga: Actually I have learnt a lot on that aspect because in terms of the **ways of solving problems**...**improve my ways of doing thing**s. Like most of the time if you look at the trigonometry… we just say to learners, "**take out your calculator, type sin 30°**" … **but what does it mean**? We don't really go into details with that as long as the learner knows how to operate the calculator.	Ways of solving problems Improve my ways of doing Meaning of sin sin 30°

Table 2- Excerpts Illustrating the Open Coding Process for Question One

Table 3 - Excerpts Illustrating the Open Coding Process for Question Two

Opportunity to use Geogebra provided the affordances to deepen her knowledge of Trigonometric Transformation. Similarly, Rolinga's main take away was also related to the ICT integration but was more focused on meaning making from the perspective of the learners. He exemplified this by explaining how he never used to draw learners attention on the meaning of certain outputs from calculator. In axial coding Thandi's use of Geogebra to overcome the challenge of performing Trig. Transformation and Rolinga's new approach of explain the meaning of outputs in calculators can be classified as actions or strategies taken to address a problem. In Thandi's case the problem to overcome was to understand a specific mathematics problem while Rolinga's concern was on developing the means of explaining the meaning of Trigonometric ratios. Overall, both teachers when addressing how their participation in the course has transformed their teaching described significant changes. Explaining the mathematics content and designing tasks that expose structure to learners are some of the things Rolinga reported as his uptake of PD. Similarly, Thandi cited becoming more aware of the purpose of example set, in addition to understanding that she does not have to dominate the classroom discussion and that, the focus of a lesson is more about what learners do as key influences of the MTF in her practice.

Raw Data	Opening Coding

Thandi: … I've learnt a lot about the teaching framework, like the use of examples, the use of... when you e**xplaining to the learners** and what is **learner participation** because before I started attending the course, I realized that I was **doing more of the talking in the classroom** than the **learners doing more of the work** or the talking. So it has helped realize that the **lesson is more about what the learners will do**… and also with **choosing examples** like in terms of setting papers. **I'm more aware of what exactly that I'm asking her**e.	Learn about the teaching framework Doing more of the talking learners doing more work (tasks) more aware of the purpose of examples
Rolinga: … Exemplification when we looking at the **issues of variance and invariance**…the things that we take for granted that learners actually understand. But those are the key aspects to make like, do you give your learners; in terms of participation in class how do you do it?	Issues of variance and invariance in example set
Why are you doing it? **What is the goal of the lesson** …? When you select the **examples**, we just **selected randomly**. You never really thought about, "why am I selecting this?"	Randomly selecting examples

Table 4 - Excerpts Illustrating the Open Coding Process for Question Three

Conclusions

The examples used to illustrate teachers' stories about take up of PD and its influence in their practices suggest that when teachers are given opportunities to reflect on their practice, in relation to how it is situated in the larger context of policy and their work environment, they are able to enhance their instructional thinking. As evident in the transcripts above, both teachers' narratives highlighted a prior need for self-development, to improve ways of doing things in their contexts where factors relating to their mathematics knowledge for teaching, their work environment and policy regulations were limiting what they could do to develop as teachers. What is interesting is both are qualified teachers with motivation to learn and change. The elements of the framework resonate with their motivations and so goes some way to explain their take-up.

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Relationship between the instructional practices and epistemic beliefs of Kenya secondary schools mathematics teachers

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Evidence from extant research suggest that teacher-centered teaching continues to be the norm in most Kenya secondary schools. Evaluation studies have largely focused on and attributed the observed persistence of teacher-centered teaching, despite training in student-centered teaching methods, to factors external to mathematics teachers. This presents a gap given the understanding that instructional practices are shaped by factors external and internal to the teacher. In this qualitative multiple case -study I use the Clarke and Hollingsworth (2002) Interconnected model of professional growth to explore the relationship between the Kenya Secondary mathematics teachers' epistemic beliefs and their instructional practices. The findings of the study suggest a dynamic relationship between the teachers' epistemic beliefs, the epistemological orientations of their trainers, the teacher training course contents and ultimately the teachers' instructional practices. Based on the findings, policy recommendations for pre and in service training of mathematics are offered.

Keywords: teacher-centered teaching, student-centered teaching, multiple case study, epistemic beliefs, instructional practices.

OVERVIEW OF MATHEMATICS TEACHING REFORMS

For more than three decades now, there has been a consistent effort amongst mathematics education researchers (cf. Kutaka et al 2017) and stakeholders to reform mathematics teaching from teacher centered to learner centered. The push is largely informed by findings which have shown a positive relationship (ibid.) between learning and active engagement of students in the learning process. Further, the constructive nature of this form of learning is also considered to be in line the current efforts being made by many global leaders to build more knowledge based economies (Chai, Hong & Teo 2009).

Kenya for example has earmarked improving the quality of teaching mathematics, science and technology in schools, polytechnics and universities as one of its key policy strategies for achieving its vision of moving to a knowledge-driven economy by the year 2030. (Government of the Republic of Kenya 2007). As a result, the Kenyan Government through its' Ministry of Education, invest millions of Kenya shillings each year (CEMESTEA 2014) in a government run in-service education and training (INSET) program for secondary mathematics and science teachers. The program titled, Strengthening of Mathematics and Science in Secondary Education (SMASSE) has its roots in Japan and is mainly geared towards shifting the pedagogical orientation of Kenya secondary school mathematics and science teachers from teacher-centered teaching to student-centered learning.

The aforementioned benefits notwithstanding, the uptake of student- centered instructional practices by mathematics teachers across the world has been somewhat slow amongst mathematics teachers. In the case of Kenya for example, evidence from empirical research found no significant difference in the instructional practices of SMASSE trained and non-SMASSE trained mathematics and science teachers (Njoka et al. 2013; Sifuna & Kaime 2007).

The multidirectional framework for professional growth of teachers developed by Clarke and Hollingsworth in 2002, provides a succinct summary and categorization of the key factors considered to influence the teachers' adoption of student-centered pedagogies.

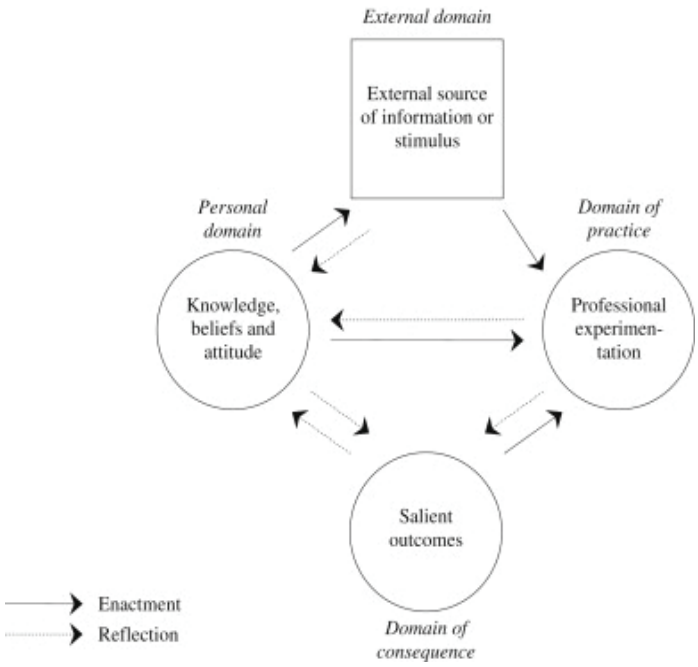

Figure 1. The Interconnected Model of Professional Growth (Clarke & Hollingsworth 2002, p.951).

As summarized in this model, long term change in instructional practice has been widely attributed to a multi-directional interaction of factors external and internal to the teacher (Voogt et al. 2011). A majority of the studies done to evaluate uptake of student-centered instructional practice by mathematics teachers, in Kenya seem to focus on external factors. As a result, there is very limited understanding on the extent of the influence of

Kenya secondary schools' mathematics teachers' attitude and beliefs towards mathematics on the reported slow uptake of student-centered teaching by Kenya mathematics teachers.

Through this study, I sought, to contribute to filling this gap by exploring: the resonance between the Kenya mathematics teachers' beliefs of mathematics and student-centered teaching; and the factors (SMASSE related or not) that may have shaped the six teachers' beliefs towards mathematics teaching and learning.

Mathematics Teachers' Beliefs and their Instructional Practices

Due to the scope and time limits for carrying out this study, I restricted myself to exploring the link between the Kenya mathematics teachers' epistemic beliefs (excluding self-efficacy beliefs) and their adoption of student centred teaching. Teacher epistemic beliefs may present themselves in a continuum, (Pepin 1999). At one end of the continuum are the instrumentalists who tend to associate mathematics as a discipline with accumulation of facts, rules and procedures and in turn embody a performance oriented content focused view of teaching and learning of mathematics (Cross 2009). At the middle of the continuum are the Platonists, while identifying with a non-discrete static view of mathematics and generally support a content oriented approach of teaching their ultimate focus tend to be students' conceptual understanding of 'the logical relations among various mathematical ideas and the concepts and logic underlying mathematics procedures' (Pepin 1999, p. 137). As a result, they tend to pay more attention to structure of mathematics over the students' interests or ideas. At the end of the continuum, are the problem solvers who are sometimes referred to as constructivist (Pepin 1999) who hold a view that mathematics is a dynamic, expanding and continuously changing discipline of knowledge. Constructivists view mathematics as a process of enquiry and 'not a finished product' and put great responsibility of learning of mathematics on the students and position the teacher as a facilitator and stimulator of students' learning of mathematics (Pepin 1999; Cross 2009).

Kenyan education context

Kenya has a tiered system of secondary schools made up of four tiers; National, Extra County, County and Sub-County schools. The distribution of students across the tiers is competitively linked to the Kenya Certificate of Primary Education (KCPE) examination results.

The participants

The participants were six mathematics teachers who were selected to represent varied experience of teaching mathematics across schools from the five tiers of secondary schools in Kenya. Ethical clearance for this study was obtained from the university.

DATA GENERATION AND ANALYSIS

The data for this study was collected through WhatsApp supported open telephone interviews and were analysed using thematics and basic discourse analysis.

FINDINGS AND DISCUSSIONS

Findings from this study, corroborated empirical evidence from extant studies (Chai, Hong & Teo 2009; Cross 2009) that point to a strong relationship between mathematics teachers' epistemic beliefs and their instructional practises. Specifically, the findings added credence to earlier findings (Brownlee 2004; Chai, Hong & Teo 2009) that have shown that teachers with a more sophisticated view (leaning more towards constructivism) are likely to be open to employing more student-centred teaching strategies in their daily teaching of mathematics. Notably, the findings suggested that the Kenya mathematics teachers epistemic beleifs may be multidimensional (Schommer 1990) in nature. There was also some evidence suggesting that contextual strains may sometimes override a teachers' epistemic beliefs in determining the instructional practises. In a resonance with findings from other studies (cf. Deng 2004), the findings suggest that the shaping of the mathematics teachers epistemic beliefs seemed to be influenced by: epistemological orientations of the pre and in-service teachers trainers; the level focus on theoretical and reflective understanding during pre and in-service training; and the teachers' mathematics intelligence beliefs; a towards mathematics during the two sets of training on developing theoretical and reflective understanding of the teachers on how students learn mathematics.

Implications

The study findings provide key pointers of aspects of teacher development that may need be taken into considerations by key Kenyan (and African) institutions such as: teacher training colleges; Universities; and policy makers/institutions in charge of pre-and in-service training of teachers.

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Mathematics teachers' understanding of smasse principles of ASEI

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Strengthening of Mathematics and Science in Secondary Education (SMASSE) In-Service Education and Training (INSET) is a program that has been implemented in Malawi since 2004 with the aim of improving the teaching and learning of mathematics and science in secondary school. Teachers are expectd to implement Activity, Student-centred, Experiment and Improvisation (ASEI) lessons in their classroom achieved through the Plan, Do, See, and Improve (PDSI) cycle. This paper reports on a study that investigated how mathematics teachers understand the SMASSE principles of ASEI. Qualitative data were collected through interviews, lesson observation and document analysis from 6 teachers, purposively selected from 3 secondary schools in South East Education Division of Malawi. Findings of the study include that teachers had limited knowledge of some of the SMASSE principles such as Activity and Student-centred. Also teachers' understanding of an activity was slightly different from what SMASSE considered as activity. However, when the mathematics teachers worked together in a Community of Practice for one year, their knowledge, understanding and practice of ASEI improved. The findings challenge the assumption that teachers can implement SMASSE principles individually after undergoing workshops, and highlight the importancs of mathematics teachers working together in a Community of Practice.

KEY WORDS: Malawi, community of practice, in-service, secondary mathematics,

SMASSE

BACKGROUND INFORMATION

SMASSE INSET Malawi is an initiative by Government of Malawi in her attempt to improve the quality of secondary school mathematics and science education. The idea originated in Kenya in1998 and introduced to Malawi in 2004, supported by Japanese International Corporation Agency (JICA) in both countries. The curriculum for both SMASSE INSET Malawi and Kenya featured Activity, Student-centred, Experiment and Improvisation (ASEI) Principles (KSTC, 2002; DTED, 2009).

Activity, Student-centred, Experiment and Improvisation (ASEI) Principles

Activity: Teachers must incorporate activities in lessons that promote development of interest in the subject, manipulative and communication skills, intellectual thinking and reasoning (hearts-on, hands-on, mouths-on and minds-on). **Student-centred:** This aspect of ASEI discourages the teacher's dominance in the teaching and learning process but rather facilitating it and encourages the learner to be the main actor in the classroom. **Experiment:** This emphasizes a shift from large scale type of experiments to small scale or investigative type where learners are allowed to make predictions, hypotheses and verify them practically. **Improvisation:** Improvisation in ASEI means adopting materials modelled from the learners' environment not necessarily due to insufficient conventional materials but for the sake of contextualization of concepts so as to raise learners' interest and curiosity.

STATEMENT OF THE PROBLEM

Research shows that Malawi is facing a lot of challenges regarding performance and participation of learners in mathematics and science at both primary and secondary school levels. Some of the reasons for this situation are lack of qualified teachers, inappropriate teaching methodologies and lack of teaching and learning resources [Domasi College of Education (DCE, 2003)]. Despite teachers being trained to teach using more learner centred methods by SMASSE INSET Malawi program, their teaching continues to rely more on 'chalk and talk' [Department of Teacher Education and Development (DTED, 2009; DTED, 2012; Nampota & Selemani-Meke, 2014)]. It was therefore worth investigating mathematics teachers' understanding of ASEI lessons.

LITERATURE REVIEW

So far SMASSE Kenya and SMASSE INSET Malawi have managed to achieve regularized and institutionalized INSET systems for mathematics and science teachers. However, they both lament that the practice of ASEI/PDSI at classroom level remains unsatisfactory (Kahare, 2011; Karuri, 2012; Kamau, Wilson, & Thinguri, 2014; DTED, 2009; DTED, 2012; DTED, 2016; Nampota & Selemani-Meke, 2014). Furthermore, DTED (2016) in Malawi specifically found that lesson evaluation done by the teachers on their own lessons showed ASEI/PDSI performance index of 3.1 while that of DTED officials on the same lessons was 2.15 against a bench mark of 2.5. This may indicate that teachers' understanding of the concept may be different from what is intended by SMASSE INSET Malawi.

Theoretical and conceptual framework

Constructivism was used as an umbrella theory of learning, while the PDSI (the vehicle for achieving ASEI lessons) concept guided the research methodology (Jaworski, 1995; Bayne & Horton, 2003; DTED, 2013).

Plan, Do, See, Improve (PDSI) approach

Plan: The teacher is expected to prepare small steps of a lesson with its activities or experiments to allow learners to follow logical flow of the lesson by themselves. **Do:** The teacher needs to conduct the lesson as planned while being innovative in the presentation and varying presentation methods to arouse interest in the learners. **See:** As the lesson progresses, the teacher is expected to observe and evaluate the teaching and learning process to check learners' progress and for the teacher's own improvement. **Improve:** Using the information in 'See' above, the teacher must reflect on their performance to see good practices to be strengthened and mistakes to be corrected or avoided. This process would also help the teacher when planning for the next lesson to improve their own practice and performance of learners thereby completing the PDSI cycle (DTED, 2013).

METHODOLOGY

The study was placed within the interpretivist paradigm and used qualitative research design to generate data through interview (one at the beginning and one at the end of the study), lesson observations (in Communities of Practice) and document analysis. The study had a sample of 6 teachers purposively selected from 3 different schools to target those trained by SMASSE and all of them were teaching mathematics.

The CoP was established as part of the project implemented by the University of Malawi with support from JICA to find out what can be incorporated in preservice education for teachers (Kazima, Mbano, & Nampota, 2015). The researcher followed teachers in the CoP as a participant observer with minimal interference in an endeavor to identify the support teachers need in order to practice ASEI/PDSI as it is intended by SMASSE INSET Malawi.

Data Analysis

Guided by the PDSI conceptual framework, data from interviews and lesson critiquing sessions were transcribed, analysed and clustered around the themes of Planning, Implementation, Reflection, Improvement and knowledge of ASEI/PDSI.

Ethical consideration

Permission sought from South East Education Division Manager prior to commencement of the study, purpose of study explained to participant & they signed a consent form and participated of their own free will.

RESULTS AND DISCUSSION

There were a lot of interesting findings from this study, but this paper will concentrate on teachers' knowledge of Activity and student-centred lessons.

Teachers' knowledge of Activity and student-centred before CoP

When teachers were asked whether they included activities for learners in their lessons, the following were some of the responses they gave:

> "I give them exercises together in groups. So when you have given them the exercise whether in pairs or as a group they can teach each other, " (Teacher A, Interview 1).
> "Sometimes just an exercise which will take us to the concept which we want the students to grasp," (Teacher C, Interview 1).
> "Yes. Like in mathematics you can give a problem and the learners to do in pairs," (Teacher D, Interview 1).

From the rest of the responses, teachers cited group work and class exercises as examples of activities learners were involved in. They would give an exercise or a problem with already known procedures for solving so that learners worked in groups. These were taken as 'Activities' for learners and as long as they were doing something, the lesson was termed as student-centred. According to ASEI, activities need to be engaging to promote development of interest in the subject, manipulative and communication skills, intellectual thinking and reasoning. Such a lesson is student-centred (DTED, 2013). It can therefore be concluded that teachers' knowledge of activity and student-centred was different from what SMASSE INSET Malawi intended.

Teachers' knowledge of Activity and student-centred in the CoP

During critiquing of lesson plans, teachers would look for more engaging activities and even write down as many expected responses from students as possible. Their perception of activity was different as it can be seen from some of the responses below when they were asked to explain their benefits from CoP:

> "Now I am writing different lesson plans. It has learners' activity and teacher's activity. I now include everything on a lesson other that the sketch that I did", (Teacher E, Interview 2).
>
> "The lesson plan we are planning involve learners' activities so which means ASEI/PDSI is inside while the previous lessons we not dwelling very much on ASEI/PDSI. We were just planning not even bothering ASEI/PDSI", (Teacher D, Interview 2).
>
> "I think the teacher needs to be assisted the way we have done with the community work with the lesson plans planning because students do not enjoy much when you give them a problem to solve in groups using already known method, they like the issues of argument. It is not easy to find good activities when you plan alone." (Teacher B, Interview 2).

In the CoP, through lesson critiquing and support from each other, teachers realized that they had not been delivering ASEI/PDSI lessons as they discovered that their activities had not been engaging to the learners and so the lessons had not been ASEI lessons. This finding agrees with DTED (2016) where teachers thought their practice of ASEI/PDSI was satisfactory while DTED officials' evaluation found the opposite on the same lessons. This means that teachers had their own version of ASEI/PDSI different from that of SMASSE INSET Malawi.

A general picture emerging from the study is that teachers need support in the form of supervision, encouragement and sharing ideas in order to deliver student-centred lessons. A CoP in this study proved to be a good source of this support. The study recommends that teachers work together in a CoP to support each other in implementing ASEI/PDSI at classroom level.

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Investigating primary school teachers' experiences in teaching mathematics using learner centred approaches in Malawi

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Related Sub-theme*:* Mathematical thinking for nurturing quality education

This paper discusses findings of a study that investigated primary school teachers' experiences in the teaching of mathematics using learner centred approaches. The study was a mixed methods design and data was generated using survey questionnaire, lesson observations, document analysis and semi-structured interviews. A total of 22 primary mathematics teachers responded to the questionnaire, and 8 of them were observed while teaching and were also interviewed. Findings of the study indicate that the perception that mathematics teachers have towards teaching mathematics using learner centred approaches does not match with their classroom practices. While mathematics teachers view learner centred approaches as good approaches that help learners to take part in their own learning, the way they used learner centred approaches in mathematics lessons did not seem to be learner centred, and did not promote mathematical thinking in learners as expected in the conceptual framework. This raises questions for mathematics teacher education on how best to educate teachers so that they put theory into practice.

INTRODUCTION

In an effort to address the many challenges that the primary education sector is encountering in Malawi, the Ministry of Education introduced an Outcome Based Education (OBE) curriculum in 2007. Among these many challenges is the underachievement of primary school learners in the areas of literacy and numeracy (Kaambankadzanja, 2012). This OBE has its foundation on learner centred approaches which put the learner at the centre of learning. The emphasis in learner centred education is not on who has passed or failed, but on having all learners succeed (Gunsaru & Kaambankadzanja, 2007). Thus, from 2007 to present, the Ministry of Education is implementing and still advocating the use of learner centred approaches as an effective way to teaching and learning.

However, despite the introduction of learner centred education, primary school learners continue to perform poorly in Mathematics. In an assessment exercise done by the Malawi Teacher Professional Development Support (MTPDS) (2010) with the aim of investigating the level of mathematics skills of children in Malawi, it was established that learners are performing at levels far below what the curriculum expects of them. Results of the Primary Achievement Sample Survey (PASS) conducted by the Ministry of Education Science and Technology (MoEST) in standards three, five and seven revealed that less than 8% of standard three learners attained the expected level of numeracy. No learner scored more than 50% in mathematics in standard five, and 99% of standard seven learners scored less than 50% in mathematics (MoEST, 2010). All this shows that learners in primary schools are not doing well in mathematics.

This poor performance in Mathematics may be an indication that, among other reasons, learner centred approaches are not being implemented the way they are supposed to be. However, very little has been done to find out teachers' experiences in the teaching of mathematics using learner centred approaches despite the fact that teachers are the ones implementing the reform, and that there is still an outcry of learner underachievement in mathematics years after the introduction of learner centred education. Thus, this paper reports on a study that sought to investigate primary school mathematics teachers' experiences in teaching mathematics using learner centred approaches, with the assumption that understanding what actually goes on in the mathematics classrooms pertaining to learner centred teaching may help in exploring more ways of supporting teachers in the implementation of active learning approaches in the teaching and learning of mathematics, thereby, improving learner performance. The following questions guided the study:

- What learner centred approaches do teachers use when teaching Mathematics?
- How do mathematics teachers use learner centred approaches in teaching Mathematics?
- What are the teachers' perceptions of learner centred approaches in the teaching of Mathematics?

Theoretical Framework

Theoretically, the study was grounded in the theory of constructivism as advocated by Piaget, Vygotsky and Bruner. In this theory, learning takes place when learners actively construct meaning rather than passively receive it (Piaget, 1977). This theory regards knowledge as explanations that are constructed by individuals as they engage in meaning-making activities in their social and cultural environment, and not as truths that can be transmitted. Thus, constructivism assumes that learners actively construct their own understanding of mathematical ideas through interaction with others as well as their environment (Cathcart et al, 2001; Du Plessis & Muzafar, 2010). The theory of constructivism gives guidance on moving away from traditional approaches of teaching to those approaches that are interactive and help learners to actively and critically create meaning, and what principles to follow to make mathematics teaching and learning learner centred. There are six principles that have to be followed if teaching and learning is to be learner centred; these include: learning as an active and interesting process, a constructive process, a situated process, a cooperative process, a reflective process, and that the teacher should be a facilitator (InWent, 2009).

Methodology

The study followed a mixed methods approach, using sequential explanatory strategy in which data collection started with quantitative approach followed by qualitative data in the second phase. Results of the quantitative phase informed the qualitative part of data collection. Twenty-two primary school mathematics teachers from five primary schools in Zomba district completed a survey questionnaire. The aim of the questionnaire was to find which mathematics teachers teach using learner centred approaches, and the results of the survey helped to purposefully select the eight teachers for lesson observation and interviews in the qualitative part of the study. Each participant was observed twice, and one interview was conducted on each. Basically, data in this study was collected through survey questionnaire, lesson observations, semi-structured interviews and document analysis.

Data was analysed both quantitatively and qualitatively. Data from questionnaires, document analysis, and some from lesson observations were statistically analysed using frequency counts, and the findings presented in form of tables and graphs to clearly show what approaches teachers use and how they understand learner centred mathematics teaching. Qualitative data was analysed inductively.

Results and Discussion

Results from questionnaire and teaching records indicated that teachers are aware of a number of learner centred teaching and learning approaches, and they claimed to be using them often in teaching mathematics. However, classroom experience revealed that very few learner centred approaches out of the many that they claimed to use were really used, i.e. demonstration & practice, discussion, question & answer, group work, observation, and role play. It was observed that some methods that are more teacher-centred than learner centred, such as explanation, dominated the lessons. This may mean that teachers have theoretical knowledge about learner centred approaches, but few are put into practice; thus they reported what they know, and not necessarily what they do.

On how teachers use learner centred approaches in teaching Mathematics, it was observed that although mathematics teachers used some learner centred approaches in their teaching, the way they used them did not seem to give learners opportunity to conduct meaningful discussions, question their thinking or make sense of their learning. Teachers mostly asked low level questions that required learners to recall concepts they already knew, and they rarely asked learners to explain their thinking. Group work was also used to a greater extent in the lessons that were observed. However, in most lessons, learners were put in groups mostly to practice what they had already discussed with their teacher in an example. Such use of group work does not agree with what the theory of constructivism requires that learners should be encouraged to construct their own understanding of concepts. The way learner centred approaches were used in this study may indicate that teachers have limited knowledge about the teaching of mathematics using learner centred approaches.

Regarding the perception of teachers in teaching mathematics using learner centred approaches, the study revealed that mathematics teachers view learner centred approaches as good approaches that help learners to take part in their own learning. They recognise the benefits that come with teaching mathematics using learner centred approaches. Almost all teachers indicated that they are very comfortable with teaching mathematics using learner centred approaches. However, their perception did not seem to match with their classroom practice. Although they seem to have enough theoretical knowledge and positive attitude toward learner centred approaches, implementation was a problem; most of their lessons seemed to be more teacher centred than learner centred.
One possible explanation for why teachers had problems implementing learner centred approaches as expected might be that during their teacher education in College, they might have learned about the approaches theoretically, but their teacher educators might not have been modelling them. For the teachers who were already teaching when learner centred education was introduced, possibly, the orientation they received from Ministry of Education might not have been adequate to educate teachers on how they could effectively use the approaches. Another possible reason could be lack of reference materials as teachers reported only using teachers' guides and learners' books for planning. Teachers' guides and learners' books only suggest approaches that teachers can use during lessons; they do not contain any explanation on how to use those approaches. Thus, having suplementary reading materials on teaching and learning approaches may help teachers to know how a particular method is used.

Thus, the findings from this study indicate that there is need for continued support to mathematics teachers as they implement learner centred teaching. The results also raise questions as to how teacher education helps teachers to effectively implement learner centred approaches in their mathematics lessons because teacher performance largely depends on the type of training they receive. Therefore, teacher educators need to model learner centred approaches in their mathematics lessons in order for their student-teachers to emulate their example and do the same with their learners.

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From quantity to quality mathematics education in Malawi: lessons from a five year project

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At AFRICME4 in 2013 in Lesotho, we presented a research proposal for improving quality of mathematics education in Malawi. The proposal was accepted by the Norwegian Programme for Capacity Building in Higher Education and Research for Development (NORHED), and we were granted funding for period 2014-2018. Five years later at AFRICME5 we present our project work over the years. Malawi has struggled to cope with large numbers of students in schools since the introduction of free primary education in 1994. Consequently, quality of education has been compromised. Our project's objective is to contribute towards improving quality of mathematics education in schools by improving quality and capacity of mathematics teacher education. Our work is informed by theories of Mathematical Knowledge for Teaching, and we take the view that the most important resource in mathematics teaching is the teacher. The project has five components; PhD programme, master programme, professional development, research, and infrastructure development. We discuss each of these and highlight the achievements as well as challenges. Finally, we discuss lessons learnt and implications to future work in Malawi or other underdeveloped countries.

Sub-Theme: Role of contextually relevant research in quality Mathematics Education

BACKGROUND

Malawi introduced free primary education in 1994. While this was a big step forward for Malawi, it also brought many challenges to the education sector. Student enrolment in primary schools almost doubled the first two years (Kazima & Mussa, 2011). The large increase in enrolment solved the problem of access to primary education for all children, but the schools did not have enough facilities and teachers. At the start of this project, the average teacher to pupil ratio in Malawi primary schools was 1:88 (Ministry of Education, 2013). In an attempt to cope with the high demand of teachers, the Malawi government employed many unqualified teachers and introduced fast track teacher education for primary schools. A consequence of this is that quality of teaching in general, and quality of teaching mathematics in particular, has been low. There are low achievement levels in Mathematics in both primary and secondary schools, as evidenced from national examinations and international assessments. For example, the Malawi National Examinations Board (MANEB) has reported less than 50% pass rates for

Malawi Schools Certificate of Education examinations for the past 10 years (MANEB, 2017). Furthermore, assessments such as the Southern and Eastern Africa Consortium for Educational Quality (SACMEQ) and the Early Grade Mathematics Assessments (EGMA), have shown that Malawi primary school children perform below the expected levels of the Malawi curriculum (Brombacher, 2011; Hungei et al., 2010). This is of concern because Mathematics is crucial for social economic development of any nation, thus it is important to pay attention to these findings and address the factors leading to such low achievements in Mathematics. One of the main factors is the low quality of teachers in primary schools (Kazima, 2014).

With this background, the improving quality and capacity of mathematics teacher education in Malawi project was implemented with the overall goal of improving quality of teaching and learning mathematics in Malawi schools through capacity building of mathematics teacher education at University of Malawi and at primary teacher education colleges. It was expected that by improving the quality of mathematics teacher education, the quality of teachers that are produced will also improve, and in the long term improve the quality of mathematics teaching in schools.

The project is informed by theory of mathematical knowledge for teaching (Ball, Phelps, & Thames, 2008) that mathematics teachers require much more than knowledge of mathematics and knowledge of pedagogy. Ball et al. (2008) and other researchers in this field, have described different categories of knowledge that mathematics teachers need to have for effective teaching; subject matter knowledge (common content knowledge, specialized content knowledge, horizon content knowledge) and pedagogical content knowledge (knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum). Learning from these studies, the project takes the view that to improve quality of mathematics teachers we have to improve the quality of mathematics education. Furthermore, to improve teacher education is to provide opportunities for student teachers to learn the different categories of knowledge that enables effective teaching.

THE PROJECT, ACHIEVEMENTS AND CHALLENGES

The project is a collaborative development project between University of Malawi and University of Stavanger and has five components, each with intended outcomes which together feed into the goal of improving quality of mathematics teacher education. The five components are (i) PhD programme, (ii) master course, (iii) professional development programme, (iv) research and (v) infrastructure development. So far most of the intended outcomes have been achieved, and we discuss these in five sections according to the components

PhD programme

Before the project, University of Malawi had no PhD programme in Mathematics Education or in Mathematical Sciences. There was, however, a general PhD programme in Education. The project designed and established a PhD programme specialising in Mathematics Education. The project also designed and implemented a PhD programme in Mathematical Sciences. Supervision of the students is done jointly between the University Malawi and the University of Stavanger. The project offered four fellowships to University of Malawi staff; two females and two males. They started in 2014 and to complete in 2018. By mid-January 2018, two had successfully defended their thesis. This has increased the number of staff with PhD in Mathematics Education at University of Malawi, thus meeting project objective. We experienced one challenge of keeping all students motivated enough to continue with their studies. One student dropped out in his third year in 2017, despite our many efforts to retain them all.

Master course

Prior to the project, the Faculty of Education at the University of Malawi had a master of education programme and some courses for mathematics and science education. However, due to limited staff, there was no specialised master programme in mathematics and science education. The faculty was short of one course to have the specialized master programme. The project developed a master course on the History and Pedagogy of Mathematics. The course was adapted from University of Stavanger and is taught by project team from University of Stavanger. This has made it possible for University of Malawi to offer the specialized master of education programme, which is an achievement for the project. So far we have recruited two cohorts; the first in 2014, and second cohort in 2016. A third cohort will commence in October 2018. In the last year of project, the History and Pedagogy of Mathematics course will be taught jointly by the University of Stavanger and the University of Malawi, in particular involving the PhD students that just completed. This is to ensure that the course will continue to be offered at University of Malawi after project life and assuring the sustainability of the master program.

Professional development programme

Mathematics teacher education in Malawi is done at two categories of institution; the University for secondary school teachers and teacher education colleges for primary school teachers. The project designed and developed a professional development programme for mathematics teacher educators for primary school. The project is working with all eight public teacher education colleges for primary schools in Malawi and offering all mathematics teacher educators a professional development course. The course started in 2016 and runs from May to November of each year; in May each year, the project offers mathematics teacher educators a three day workshop where they are introduced to lesson study and concept study in mathematics. Between May and November, each college conducts lesson study and video records the lesson as well as their discussion of the lesson. In November there is a follow up workshop where the teacher education colleges report the work and lesson study they have conducted, and also the discussions and what they have learnt from the lesson study about teaching and learning mathematics. The workshop also discusses a mathematical discourse in instruction (MDI) framework (Adler & Ronda, 2017) that teacher educators can use to study and evaluate their mathematics lessons, resource books and textbooks.

This professional development component has so far achieved project objectives of having all mathematics teacher educators recruited into the professional development course. The main challenge in this component is that there are very few female mathematics educators in the colleges, such that our target of reaching at least 30% female is not possible.

Research

Research in the project is integrated with all the other activities. There are two categories of research; by postgraduate students – doctoral and master students, and by project team – collaboration between the two universities. So far there have been a total of 22 research studies in mathematics education; 3 PhD studies, 15 master studies, and 4 collaborative studies by project team members. The studies have addressed issues of teaching and learning mathematics in both primary and secondary schools. The findings of these studies have greatly informed the project and the mathematics education community.

Infrastructure development

This is the last component, which is taking place in the final year of project – 2018. It is for capacity building for University of Malawi. This has focused on developing a mathematics room to be used by University of Malawi for teaching and learning mathematics education and mathematics. It has come in the final year of project because we waited for research findings and professional development to inform project on what to include in the mathematics room. The mathematics room will be an important achievement for project because it will greatly build the capacity of University of Malawi in teaching pre-service and in-service mathematics teachers. The room will add to sustainability of project outcomes after project life.

LESSONS LEARNT AND SUGGESTIONS FOR FURTHER WORK

The project has learnt many lessons from the various research and experiences in the five components. Probably the most important lesson is that attempts to change practice can be challenging if the context is not well understood. For example, while project encourages student participation in mathematics classrooms and the need for teachers to plan for activities that will get students to participate in mathematics lessons, we have to understand what would count as student participation in Malawi classrooms with very large numbers of students. To improve practice in such contexts requires careful examination of the modern ways of teaching and modifying to suit the context. It is also important to examine the aspects that work in the traditional ways of teaching and merge these with the modern ways. This is something that project team is taking up into a new NORHED project titled: *strengthening numeracy in early years of primary education through professional development of teachers in Malawi.*

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Investigating mathematics teacher learning when using a research-designed resource in a lessoning study

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This paper draws from an ongoing research study which is aimed at exploring how preservice mathematics teachers learn from and with the research-designed tool, The Mathematical Teachers' Framework (MTF) in a particular context, a 'lessoning study'-(a model with features of Japanese Lesson study Swedish Learning study). This aim is achieved through the lens of a developing analytical framework based on Wenger's (1998) notion of meaning and practice within a community of practice (CoP).

BACKGROUND

This study fits largely within the studies on research-designed resources (e.g. Kieran; Tanguay & Solares, 2012). The MTF tool was developed within the Wits Maths Connect Secondary Project (WMCS)[10] as a discursive resource, based on identification that there were some key needs and challenges in mathematics teaching in South Africa (Adler & Ronda, 2015). Adler and Ronda (2015) developed the MDI framework to illustrate how a specific object of learning (OoL) can be dealt with, in bringing about opportunities for student learning. The capability that teachers want learners to develop is brought to focus via three key elements of teaching: exemplification; explanatory talk and learner participation as illustrated in the figure 1 below:

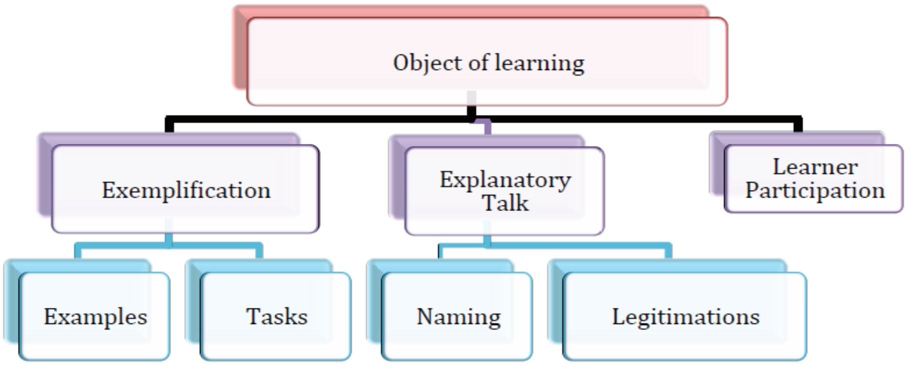

Figure 1: MDI framework (adapted in Adler and Ronda (2015, pg. 3))

[10] The (WMCS) Project is a five-year research and development project, working in collaboration with the Gauteng Department of Education at district and provincial level, the project supports teachers from 11 secondary schools in one district in the Johannesburg area. A central goal of the WMCS project is to improve the teaching and learning of mathematics within each school through an ongoing professional development (PD).

The MDI reflects the following key aspects that are reflecting the socio-cultural underpinnings and re-contextualizing of other theoretical resources: Object of learning; examples and tasks; naming and legitimating; interactional patterns (learner participation). It is these concepts that inform the MTF tool and so are key to teachers' evolving meaning and practice in and through this study.

In this study I argue for the lessoning study (hybrid of Japanese lesson study and Swedish learning study and adapt these into a South African context) as an alternative platform to work within the research-practice gap. Hence the significance of this study should be understood in that context. From Wenger's (1998) social learning perspective, I argue for "a lessoning study model" as an instance of a boundary encounter between researchers and teachers; and as a premier space for researchers and teachers, impacting both communities of practice. Kazemi and Hubbard (2008) have questioned the role of researcher in the classroom context. They pleaded for future research to capture the interactions between researchers and teachers as they collaboratively exchange knowledge in a specific context. The main contribution of this study is the development of a framework for describing the process where researchers and teachers gather and share a resource to achieve goals of both communities. Furthermore, this study is highlighting a role of a researcher within a research-practice context and the results of this study will provide insights into preservice mathematics teachers' learning (through participation in a form of CoP-lessoning study, structured to enhance participation) in relation to use of a research-designed tool-MTF.

METHODOLOGY

Data collection

The group of four teachers and myself (researcher) formed a lessoning study group (Community of Practice-CoP) collaboratively working together on a shared lesson plan for teaching a topic of choice within grade 10 function topic, with the guidance of MTF tool. Through the lessoning study cycles, the CoP members were afforded opportunities to reflect on the best way to handle the objects of learning. The aim of lessoning study model is to create innovative learning environments with a research-designed resource. As such, it is aimed at pooling teachers' valuable experiences in one or a series of lessons to improve their teaching and learning (Marton & Pang, 2006).

The data was collected from different kinds of instruments (e.g. observations, field notes-researcher's journals, and teachers' reflective journals) as they provided different contexts and produced multiple discourses. Three lessoning study cycles were conducted. There were three sessions in each cycle of the lessoning study: 1.The lesson planning; 2.

Teaching, observing, debriefing (reflective discussions); and 3. Re-teaching. The lessoning study cycle is depicted in the figure 2 below:

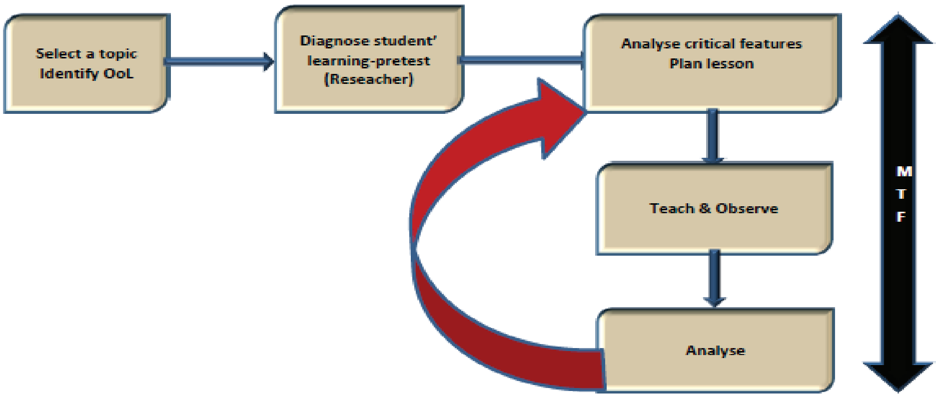

Figure 2: Lessoning study cycle

Data analysis

In this study teacher learning is described in terms of Wenger's (1998) concept of negotiation of meaning: duality between reification and participation. Secondly, teacher learning is also described in terms of how a research designed tool-MTF is used in the lessoning study activities/practices, (i.e. in terms of reflections and keeping focus on Ool-critical aspects of the OoL).

Research Questions	TEACHER LEARNING	STAGES OF L/S	MARKERS
1.What meanings do teachers attach to MTF?	***Negotiation of meaning***	Lesson planning	Reification -participation
		teaching	Reification -participation
		reflective discussion	Reification -participation
2. How can teacher learning be described in terms of teachers' practices?	***Focus on the OoL***	Lesson planning	Potential CA (intended OoL)
		teaching	Discovered CA (enacted OoL)
		reflective discussion	Real CA (Existed OoL)
	Reflections	Lesson planning	Reflection-for-practice
		Teaching	Reflection-in-practice
		Reflective discussion	Reflection-on-practice

During the coding stage I have found studies concerning teacher learning helpful, which consists of the following: negotiation of meaning with a reified tool (Wenger, 1998; Pepin et.al, 2013), teacher learning in a learning study context-Focus on object of learning (Runesson, 2013; Pillay, 2013), and teacher learning in a lesson study context- Teacher reflection (Posthuma, 2012). The ideas suggested in these studies were used to generate codes in the transcripts.

CONCLUDING REMARKS

This study is at the stage of generating codes and refining them. In the presentation I will present some preliminary data analysis and findings of the first cycle of the lessoning study.

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Any opinion, finding and conclusion or recommendation expressed in this material is that of the author and the NRF does not accept any liability in this regard.

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An international development intervention in mathematics education in Tanzania: looking back 25 years later.

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Informed by the research of the first author for his doctoral thesis "Cooperation, Complexity and Adaptation: Higher Education capacity initiatives in international development assistance programmes in sub-Saharan Africa" (2018), this paper examines Irish Aid funded initiatives in mathematics education (at primary, post-primary and third levels) in Tanzania in the 1990s. Testimonies of key informants (both Tanzanian and Irish) involved in these initiatives, along with unpublished archival material from the period, provide the evidence base for the qualitative research. The research methodology employed was inductive analysis and purposive sampling. The second author worked on a project supporting mathematics (and mathematics education) in both the university and NGO sectors in Tanzania from 1991 to 1994. The authors argue that, in a development context, initial and continuing mathematics teacher education requires a whole-of-systems perspective, considered over an extended period of time and transcending the boundaries between levels of education.

INTRODUCTION

In 1973, around the time of Ireland's accession to the (then) European Economic Community, the Government of Ireland initiated a programme of overseas development assistance, which later came to be known as Irish Aid (Murphy, 2012). In its early days, Irish Aid considered its comparative advantage to lie in supporting technical assistance and human capacity development projects (as distinct from large infrastructural and capital-intensive ones). The intention was to address identified skills deficits in the four chosen 'priority' countries of sub-Saharan Africa (Lesotho, Sudan, Tanzania and Zambia), drawing on appropriate Irish expertise and in response to requests from host Governments and institutions (Murphy, 2012, p. 68).

When it came to translating these aspirations into practice in the institutions of higher education in Ireland, a cross-institutional implementation body, Higher Education for Development Cooperation (HEDCO) was set up in 1975, with the purpose of acting as Irish Aid's promotional and implementing arm for higher education cooperation with Africa, through the participation of the all-island university sector in Irish Aid.

In the 1990s, HEDCO designed and implemented two projects relating to mathematics and mathematics education, one in the University of Dar es Salaam (USDM), the other in

Korogwe Teachers' College (KTC). Between these two initiatives, a rich alliance was forged between Tanzania and Ireland involving expertise and building capacity in mathematics and mathematics education at primary and secondary (post-primary) levels, as well as in initial teacher education and higher education. In KTC, the project sought to contribute to upgrade the quality of mathematics teaching in Tanzania through design, piloting and adaptation of appropriate methodologies and materials for teaching of mathematics at primary level, with particular emphasis on the girl child and the woman teacher. In UDSM, the project was designed to secure an academically self-sustaining Department of Mathematics to enable it to fulfil its University mandate for teaching and research, and to support outreach initiatives aimed at improving quality of mathematics teaching in schools and colleges.

The first author, having spent a decade of his earlier career with HEDCO, recently completed his doctoral thesis (McEvoy, 2018) examining the role of higher education capacity development as a component of international development assistance programming to Africa, provided by international finance institutions, and by OECD member states (including Ireland). The second author completed his doctorate in numerical analysis in 1984 and lectured in mathematics from 1981 until 2016, including three years in UDSM (1991-1994). Soon after arriving in Tanzania he became involved in the Mathematical Association of Tanzania (MAT/CHAHITA), developing there an interest in mathematics education which deepening on his return to Ireland. In 2016, he made the transition from mathematics to mathematics education at the DCU Institute of Education. His experience in Tanzania, along with his work in mathematics and mathematics education informs the discussion below seeing the first author's research from the perspective of mathematics education.

RESEARCH SCOPE AND METHODOLOGY

The research (McEvoy, 2018) set out to examine the historical pathways which have supported aid-funded higher education capacity initiatives (AFHECIs), and their contribution to strengthening sub-Saharan Africa's higher education systems and to wider societal transformation. It examines two public policy domains (international development and higher education) each of which is demonstrably complex and multi-faceted.

The inter-disciplinary and trans-disciplinary character of the study required a heterogeneous methodology encompassing multiple sources of qualitative evidence: (i) literature review, (ii) 27 key informant 'depth interviews', (iii) case studies using archival material (including that relating to HEDCO in general and the two Mathematics education projects in particular), and (iv) practitioner reflection spanning some 30 years of professional practice in development programme management and evaluation.

The underpinning theoretical perspective was that of Complex Adaptive Systems (CAS) theory, which has been gaining currency as a theoretical prism on topical problems in public management and organisational analysis (Land, Hauck & Baser, 2009). Consistent with the inductive nature of this study, purposive sampling, a non-random technique widely used in qualitative research for the identification and selection of information-rich subjects (Patton, 2002), was used to determine the population of potential interviewees.

The study examined critically the adequacy of the conventional techniques used by bilateral and multilateral donor agencies in assessing what constitutes an effective AFHECI. A synthesis of the different sources of evidence yielded a set of eleven constitutive attributes of effective AFHECIs, some of which are discussed further in the next section, with reference to the two mathematics education projects.

HOW THE RESEARCH RELATES TO MATHEMATICS EDUCATION

In the past decade significant attention has been paid to supporting capacity building in mathematics education in developing/low-income countries. This is especially evident in the initiatives supported by the International Commission on Mathematical Instruction (ICMI) and, in particular, the Capacity and Networking Project (CANP) which arose in response to the UNESCO report, *Challenges in basic mathematics education* (2012). This report emphasises the vital importance of collaboration, not just North-South cooperation, but also regional cooperation (p. 37). AFRICME 5 is a manifestation of the latter; we will return to the former below. The report also includes (UNESCO, 2012, pp. 73-75) an annex on research on teacher education in South Africa and Southern Africa, giving a synopsis of work (at that time) in recruitment and retention of mathematics teachers, in the selection of content for initial teacher education, and in ongoing professional development. These are global challenges, but are particularly acute in the Global South. To make progress with the most urgent demands facing mathematics education it is necessary to pay attention to building and sustaining communities to collaborate in research in the field. An engaging account of how an individual (ICMI President, Jill Adler) can inspire and foster such collaboration is given by Graven, Phakeng & Nyabanyaba (2016).

Another aspect of the terrain considered in the UNESCO report (pp. 29-30) is the goal of achieving synergy "among a variety of experts such as mathematicians, teachers, teacher trainers and educationists in particular." The complexity and diversity of the relationships between practitioners of the disciplines of mathematics and mathematics education are elaborated in a collection of essays edited by Fried and Dreyfus (2014). The many and urgent questions that arise in this discourse cannot be ignored when considering mathematics education in a Global Southern context; and yet, scarce resources and demographic pressures demand cognisance of economic constraints without compromising a spirit of generosity.

It is difficult to embrace all the contexts in which AFHECIs reside, and to draw some insights from experiences of over two decades ago of mathematics and mathematics education in Tanzania. The contexts outlined in the previous two paragraphs were then only 'in gestation'. In analyzing interviews with 27 informants, McEvoy (2018) coded 532 references in eleven categories (or attributes), five of which gave rise to the (post-pilot) conceptual framework (pp. 103-104, 189-190). Here we mention one relevant key finding from each of four of these five (with the **name of the attribute** in bold). Under **forging alliances**, it was found that "authentic partnerships need to be mutually respectful, genuinely needs-responsive and focused on institutional-level capacity," while (under **adaptation to change**) there was a "constant balancing act demanded between operating in a fluid global environment, while also maintaining strong collegiality, consensual decision-making, inclusiveness and impartiality." These findings underscore how fruitful collaboration can withstand the contingencies that arise in implementing AFHECIs. Moreover, under **purpose & motivation**, informants affirmed that "clarity of purpose is essential for AFHECIs to be effective." All of this rings very true to the second author as he recalls the day-to-day activity (25 years ago) of teaching analysis, numerical analysis or algebra to prospective teachers and engineers, identifying candidates for further study in Ireland, working with teachers in the field, ensuring textbooks were brought to publication, or visiting schools. Under **knowledge & skills**, informants drew attention to the rise of donor aversion "to deploying technical assistance (once considered key to [capacity development])." This policy shift, a trend evident in the early 1990s, was well established by 2000, but without being explicitly supported by convincing evidence.

There were other, more specific, insights arising from interviews with informants, of which we mention three (McEvoy, 2018, pp. 143-145). An independent evaluator of the KTC project remarked that a ten-year funding horizon in needed to support capacity development in the education sector. A Tanzanian lecturer with extensive experience in higher education in Ireland drew attention to the low level of awareness of quality assurance in the Tanzanian higher education sector. An Irish technical assistant (in initial teacher education) noted a reluctance to fill senior vacant posts in Tanzania by well qualified applicants from neighbouring countries.

CONCLUSIONS

The landscape of mathematics education is an extensive one. Each country has its own distinctive characteristics moulded by policy, curriculum and other official and societal norms – see OReilly, Dooley, Oldham & Shiels (2017) for an overview of the Irish one. Yet each national and regional landscape is embedded in a global one where scholars exchange ideas and learn from the global community. Considering also the analogous landscape of mathematics, this has another character, with its own practices and societal norms. These two global landscapes interact in a fascinating topology! Adding another perspective of capacity building in a world where inequality between the Global South and North has deep historical roots; now, this space is truly complicated and needs to be seen as a whole. It seems that sustained collaborative work in building communities of practice to address the challenges in basic mathematics education is already bearing fruit. Without making any extravagant claims, it also seems that some of these essential elements of collaboration were already established a quarter of a century ago on a small scale between Tanzania and Ireland.

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Issues and trends in current math classrooms

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With the current generation of students in the mathematics classrooms, there are vast differences in the study habits, skills, and attitudes toward learning. The challenge being that these students seems to be inadequately prepared for the college education. A growing concern that educators are experiencing is the trend of entitlement, request for bonuses to boost poor grades, setting of low goals for passing class, anxiety, and struggles in accommodating and embracing new approaches to learning mathematics. In this talk, we present our experiences from across international perspectives, and invite discussion on tensions in teaching millennial students. Our presentation will include commonalities and differences in classroom issues and trends among elementary school, middle school, high school, college math majors and preservice education majors.

Introduction

Around the globe, each generation of students we see in our classroom brings their own set of challenges and unique traits. This paper presents an introductory summary of the current group of students in our classes, typically referred to as the millennial generation, and how they perceive the teaching and learning of mathematics. In addition, we discuss concerns that educators are facing in the 21st century mathematics classrooms.

Issues and Trends in Teaching and Learning of Mathematics

The concerns and issues plaguing teaching and learning of mathematics are many and vary in nature ranging from society, technology, parents, educators… but mainly on students behavior, attitudes, expectations and competences. Herein, we share a few of the existing issues and trends.

In the United States and Sub-Saharan Africa, we see students drop math classes, receive poor grades, do not attend classes, and overall, do not take college education seriously. For example, when students begin their freshman year, many struggle on how to collect and organize information. They have a difficult time taking notes and organizing them (Jairam & Kiewra, 2009), despite note taking being a critical skill for students. Ideally, how can students possibly select key ideas when they miss or show up late for class and daydream or is distracted by cellphone messaging? Well, with the inundation of technology, social media, and other distractions, students have an increasing impairment to focus on their studies. We notice that as a result of these distractions, students are not prepared for the study caliber needed for college success.

On the other hand, the educational systems in both developing and developed countries promote the use of Information, Communication and Technology (ICT) in classrooms. As such regulation on usage of classroom mobile technologies is indispensable.

In Sub-Saharan Africa, the use of ICT remains a challenge because of unavailability of computers, labs and Internet, inexperience of their educators in the use of the computers, rapid changes in technology and improving instructional methods, which are mainly lecture and assignments. Its high time mathematics educators even aimed at digitizing their course contents. Policies can be made yet most educators are not supervised.

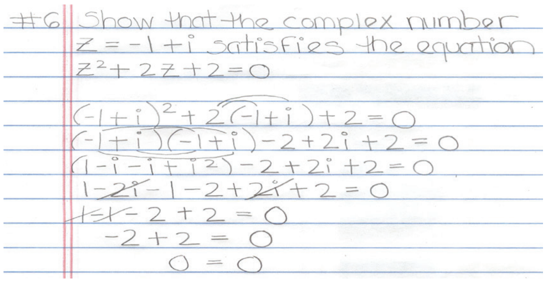
We do, however, believe that the students of the millennial generation are just as smart and willing to work as previous generations. These students are very confident and highly optimistic yet they have unassertive commitment to assignments and are stunned and quickly become frustrated when they do not achieve A or B grade in them. Our college students, whom the parents, teachers, community and the system have called "special," do not believe they can fail in anything at the university level. They get surprised that they have gotten bad grades and scores in not only examinations, but also in their teaching practice.

This may arise due to active parental involvement in these students' academic lives in their formative education. These are children who have grown up with everything being at the push of a button provided by the parents. Many of these parents are *helicopter* parents—parents who sheltered their children from failure before and cannot do so now at the college level of education. As such these students have a fixed mindset that since they got A's in elementary, middle or high school, this outstanding track record will continue into college, effortlessly.

Uworwabayeho (2009) highlight that in Rwanda, students have poor attitudes toward mathematics. This study partially attributes students' poor perception of math to teacher-centered approaches that are dominated by chalk-and-talk lectures on procedural understanding of mathematics. These emphasized rote learning, in turn leads to poor mathematics performance and to the development of negative attitudes.

Often times in mathematics classrooms, we ask students to show us a solution. This solution could be a computation, a simplification, or to show us a proof. The wording of how questions are phrase has not changed, yet the way students interpret the task directions have changed. It is a growing trend that students will write down the steps to solve a problem but not to actually do the mathematics. Explaining how to do a problem, performing calculations and giving logical arguments are essential ingredients to showing a solution but millennial students struggle to do them. Figure 1, shows misuse of equal sign in circular argument.

Figure 1



One of the challenges faced in elementary teacher preparation in the US, is anxiety that a majority of the preservice teachers have towards mathematics. Teaching PK-6 preservice teachers (PST) requires the promotion of creativity, problem solving, making connections and use of real life contexts in classrooms (National Council of Teachers of Mathematics, 2014). However, majority of the preservice teachers in their formative years of education, learned procedural mathematics—solving mathematics problems using meaningless step-by-step method, rote memorization of formulas and concepts, and algorithms (Hiebert, 2013). This difference in the learning to teach during teacher preparation and past experiences of learning mathematics creates challenges for the teacher educator and the preservice teachers.

In Kenya, the major challenge adolescent PSTs have is in communicating what they know. They struggle with the language and sometimes the anxiety comes with addressing groups of learners and other adults. In their anxiety they may say less, more, mix-up facts or fail to recall what they had prepared to teach. Some of them at this stage resort to rote memorization.

Also in Kenya, many students are absorbed in earning a good grade or score in any assessment at any cost—working very hard and going the extra mile to study which is good; cheating in exam; plagiarizing; coercing teacher to ignore mistakes by giving excuses; etc. For example adolescent preservice teachers are visited at least three times by different educators during the teaching practice semester for classroom observation followed by a debriefing meeting on the observed lesson. These PSTs are given points on many aspects of their teaching listed on the observation protocol that covers preparedness, lesson introduction, lesson development, lesson conclusion, resources, and professionalism e.g. clear and measurable objectives, arousal of interest, active learner involvement, management, recap and so on. During debriefing when the preservice teachers are asked to state what they did well and what they would like to improve on, they respond to the former. These PSTs dwell on what they did well only. Majority of these teachers are motivated by a high score as opposed to comments on how to improve on areas of weaknesses i.e. a score of 4 out 4 on "active learner involvement" even when the PST is the one who spoke most of the time with utmost two students responding to questions posed by the teacher. They rationalize why they deserve a high grade—learners are slow or struggles; the textbook has it scripted that way; curriculum or host teacher says so; no time for partner or small group work; no technology to use…

Way Forward in Teaching and Learning of Mathematics to Millennial Students

Due to page limitations, we have highlighted above a few issues and concerns noted with the millennial generation. Similarly, we feature a few strategies employed so far.

An important non-cognitive skill needed to for success is possession of a positive attitude. According to Dweck (2010), possession of a growth mindset as opposed to fixed, is what leads to success. As such, we attempt to incorporate a growth mindset in the classroom.

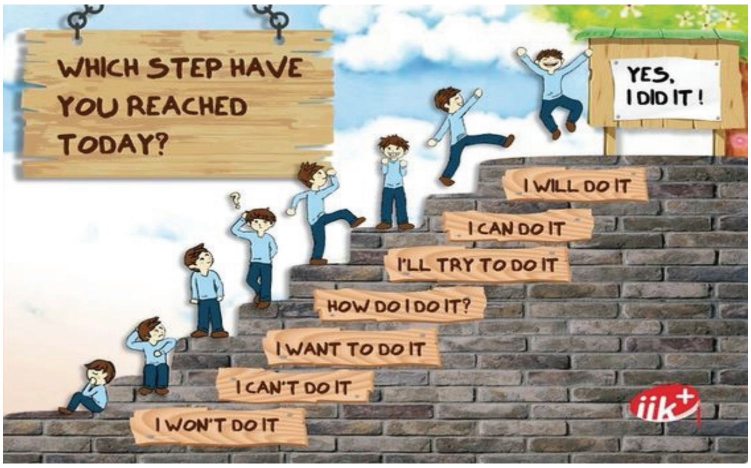
Dweck (2010) study highlights the importance of students, teachers, and principals having a growth mindset. Her quasi-experimental study conduct with hundreds of 7th-9th grade students found that students with a growth frame of mind attain more academic success than the students who have a fixed mind-set. Likewise, teachers who believe students had fixed intelligence were not able to make any changes to students who were underachievers in their class and they remained underachievers at the end of the year as compared to teachers with a progressive mind-set where the students excelled in their classes.

For the problem in Figure 1, the student should have showed the argument by writing out algebraic steps that lead to the result of zero as shown below:

$$
\begin{aligned}
&(-1 + i)^2 + 2(-1 + i) + 2 \\
&= (-1 + i)(-1 + i) + 2(-1 + i) + 2 \\
&= (1 - i - i + i^2) - 2 + 2i + 2 \\
&= 1 - 2i - 1 - 2 + 2i + 2 \\
&= -1 - 1 - 2 + 2 \\
&= -2 + 2 \\
&= 0
\end{aligned}
$$

In order to deal with PSTs' anxiety, we have encouraged reflection, introduced a positive mindset in their learning, engaged them in collaborative projects, and orchestrated in-depth discussions on selected topical areas of mathematics that have been shown to be problematic to teach and challenging to learn. This strategy has built their resilience in problem solving. Figure 2 is a representation used to facilitate reflection and to promote a positive growth mindset.

Figur
e 2



Focused reflection of PSTs' present and future self-concept

Journal writing after field observations and readings is another way for PST to reflect on their practice to improve on their teaching career and professionalism as opposed to focusing on a grade, points and scores. This process of journaling is an opportunity for the PSTs to use their developing teacher identities to analyze teaching observed in field experiences and video clips provided in some of the class sessions, and selected readings.

Additionally, providing more teaching opportunities during teacher preparation can assist in building communication skills and classroom management skills.

In concluding, there is a gap between how educators expect students to learn and how students actually learn. To close the gap, students have to first realize the differences and then take control of their own learning. Second, students have to be equipped with effective learning strategies. Lessons and the curricula must be designed with the use of technology that will not further inhibit students to learn. Parents also, must take an active and appropriate role to make sure their child is on the right path to success.

Given that this paper is an introduction to issues and trends facing the mathematics education, we hope to expand on this work. As such as we go forward, what are some ways that you have tried to support math students and preservice teachers to have more success in engaging in problem solving and expanding their mathematics knowledge that meets the 21st century competencies?

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Teachers' experiencing of one component of professional development: what does it mean?

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In this paper we report initial results from an ongoing phenomenological study that seeks to understand the meanings teachers attach to their participation in the Wits Maths Connect Secondary project's 10-year teacher professional development programme. In the full paper that we will present at the AFRICME 5 conference, we will present results from the initial survey involving 63 mathematics teachers. Sixty-three mathematics teachers were requested to complete a questionnaire at the end of the Teaching Mathematics 1 (TM1) course. The goal of the Teaching Mathematics 1 course was to deepen teachers' mathematical knowledge for teaching. Data analysis was conducted with theoretical tools drawn from phenomenology. Results suggests that teacher participation in the TM1 course may be associated with growth in teachers' understanding of selected school mathematics content and change in their perception of good teaching

INTRODUCTION

Professional development (PD) is critical for improving teachers' capacity to provide the teaching quality we so much desire for our children. Research has shown that teaching quality is related to learner achievement (Fishman, Marx, Best, & Tal, 2003; Harris & Sass, 2011). There is a tendency for teacher professional developers to assume that teachers have learnt what was made possible to learn in the PD. However, drawing from Variation theorists (e.g Ling Lo, 2012), and the duality of the object of learning in mathematics teacher education (Adler & Davis, 2006) provision of PD does not necessarily lead to teachers learning what was desired. This is because the enacted and the lived object of learning do not always cohere particularly when object of learning is dual (i.e. both mathematics and teaching) in teacher learning. Moreover, if teachers do learn, there is no guarantee that the new innovations will become part of the teacher's pedagogical repertoire in the classroom. Thus, it is imperative that any PD is subjected to some evaluation to explore whether its goal of developing teachers has been realized. It is also critical to understand whether the PD was beneficial to participants. Thus, in this paper we seek to explore the teacher's experiencing of the WMCS project's Teaching Mathematics 1 course. The particular question of interest was posed as follows: *What does it mean to experience the Teaching Mathematics 1 course*? As we were concerned with the teachers' experiences, we drew our theoretical resources from phenomenological research field which has more productive tools in handling questions about notions of experiencing and experiences (Creswell, 1994). We worked with the notions of *intentionality of conscience, horizonalization, meaning structure* and *essential meaning structure* (see Creswell, 1994). The notion of intentionality of conscience proposes that what our talk foregrounds is the focus of our minds and so is true for us at the time when it is in focus. This notion helped us to trust and believe that the teachers' responses to our survey were what they had intended. Horizonalization refers to disaggregation of the original data into analyzable pieces. The meaning structure refers to the pieces of data which may be put together to form themes. The essential meaning structure is the sum of all emergent themes and it is what it means to experience some phenomenon (Creswell, 1994).

CONTEXT OF THE SUDY

The Wits Maths Connect Secondary project is one of several continuous teacher professional development initiatives currently taking place in South Africa. Its duration is 10 years and it is presently in its 9th year. The goal of the PD is to help teachers revisit, deepen and extend mathematics in the curriculum (Pournara, 2008, 2013) to facilitate improved teaching quality in the classroom. The WMCS project's TM1 course is a 1 year long program comprising of 8 contact sessions, with each session two full days. The TM1 activities have two foci. The first, and major focus (75%) is mathematics and the second (25%) the teaching of mathematics with particular focus on the Mathematics Teaching Framework (MTF). The MTF is a theoretical resource which was developed in the WMCS as a tool for enhancing the quality of mathematics made available to learn in the classroom and has close relations with the documented Mathematical Discourse in Instruction (MDI) (see Adler, 2017). This development was necessitated by the demand for improved mathematics teaching quality in previously disadvantaged schools by funders.

METHODOLOGY

The population for our larger study is over 200 teachers who have participated in various components of the WMCS project between 2010 and 2018, the major one of which is the TM1. In this paper we draw from data from two cohorts who participated in the TM1 course in 2016 (30 teachers) and 2017 (33 teachers) to serve as the initial study to inform and shape the ongoing larger study. A survey involving 63 Grade 8 – 10 level mathematics teachers was conducted at the end of their course. All participants responded to a 13 item questionnaire. The survey sought to tap into teachers' experiencing of the Teaching Mathematics 1 (TM1) course as a way to evaluate the benefits, if there were any, of professional development offered to teachers by the WMCS project. As discussed above, data analysis was conducted with an analytical tool developed with theoretical resources from phenomenology.

RESULTS

Table 1 and Table 2 contain three columns. The first column shows script numbers. These are used to identify scripts with particular data that produced the meaning structures. The scripts were numbered from 1 to 63 for ethical considerations. However, numbering scripts in this way aided our data analysis. The second column contains meaning structures that emerged after pulling together pieces of data from different sources containing similar meanings. Outside phenomenological research these might constitute sub-themes. The third column contains the theme. In the discussion section below the themes are drawn together to describe the essential meaning structure to answer the focal question: *What does it mean to participate in the TM1 course*?

Table 1 shows that there were 13 scripts from which 7 meaning structures were developed and that together the meaning structures produced one theme. What is common about the meaning structures is their essence. Each of the seven points informs that something happened to teachers' knowledge of mathematics in the TM1 course. Now, in Table 1, any reference to teachers' own mathematics per se appears in bold and that which has happened to this mathematics is underlined. Reference to mathematics is evident in words that include: knowledge of functions; confidence (in mathematics); knowledge of trigonometry; algebra and functions; and mathematical knowledge. In each meaning structure teachers suggest that something positive happened to their mathematical knowledge. This is evident in the use of words that include: improved; clarified; understand; and rejuvenated.

Script numbers	Meaning structures	Themes
5; 6; 7; 12; 13; 16; 17; 22; 23; 25; 29; 32;33;	My **knowledge of functions** has improved **Difficult topics** were clarified My **confidence** has improved My **knowledge of trigonometry** has improved	Growth in teachers' understanding of mathematics per se

	I understand the link between **algebra** and **functions** I have developed understanding of **mathematics** My **mathematics knowledge** is rejuvenated	

Table 1: Teachers' experiencing of TM1 in relation to mathematics per se

Similarly, Table 2 above has the same three columns as Table 1. The difference between the two is what is foregrounded in the meaning structures. It can be noticed through underlined and bold words that teachers are saying something positive about their teaching per se. Words used include: teaching; my learners; selection of examples; justification in a lesson; mathematical language; and taught mathematics. The use of words such as confident; deepened and extended; more careful and patient; and improved, suggest that something positive happened to the teachers' capacity to teach.

Script numbers	Meaning structures	Themes
1; 2; 3; 4; 5; 7; 8; 9; 10; 11; 12; 13; 14; 17; 19; 20; 21; 23; 25; 27; 28; 29; 30;32; 33	I am more confident in my **teaching** The course has deepened and extended my **teaching** I am now more careful and patient with **my learners**. My **selection of examples** has improved I understand that **justification** is important **in a lesson** My use of **mathematical language** has improved I **taught mathematics**	Change in teachers' perceptions of their ability to teach mathematics

	without <u>understanding</u> before. Now my knowledge has <u>grown</u>

Table 2: Teachers' experiencing of TM1 in relation to teaching mathematics

CONCLUSION

Returning to the question of interest: *What does it mean to participate in TM1 course?*: Bringing the two themes together to constitute the essential meaning structure of teachers' participation we may assert that participation in the TM1 course increases opportunities for teachers to experience growth in their mathematics and this is evidenced by increased confidence; improved understanding of school mathematics and rejuvenation of their mathematics. Participation also facilitates change in teachers' perception of what is needed to improve teaching quality in the classroom. Knowledge of mathematics is critical for improved teaching quality and the two have strong associations with improved learner achievement. It appears what was intended is experienced. Questions of course remain as to what these are in more detail, particularly in the orientation to mathematics, and key teaching practices and their enactments

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An investigation into high school students' emerging mathematical identities

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Using an interpretive framework by Cobb et al., (2009), 34 high school students' understandings and valuations of their normative classroom obligations were examined. Data were obtained using a semi-structured interview guide and a self-reflection instrument. In order to provide an in-depth illustration of how students' understandings and valuations of their normative classroom obligations influenced their personal identities, two cases were selected using purposive sampling. Despite these two students positioning themselves differently with respect to the learning of mathematics their identity narratives provide further empirical support that the identities students form are complex and multifaceted. Also, their accounts and that of the whole group of students indicate a complex interplay between the classroom micro-culture and students' self-identifications and the varied consequences it can have on their participation levels in mathematics. Implications for mathematics teaching and learning are discussed.

Key words: Normative identities personal identities mathematics

This abstract relates to Theme 2 – Quality Mathematics Education for All

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